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Algebraic Thinking and Habits of Mind: A Systematic Review of Conceptual Foundations and Pedagogical Challenges

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Abstract: Algebraic thinking, a cornerstone of mathematical literacy, supports learners in recognizing patterns, articulating relationships, and forming generalizations across representations. However, persistent challenges—particularly in understanding variables, equivalence, and structural relationships—reveal that its development requires more than procedural fluency. Emerging research highlights the role of algebraic habits of mind—cognitive dispositions such as doing and undoing, building rules to represent functions, and abstracting from computation—in fostering flexible reasoning about mathematical structures. This study presents a systematic review conducted within the TÜBİTAK 1001 Project (No. 223K317), synthesizing 41 empirical and theoretical studies published between 1990 and 2024. Following the updated PRISMA 2020 guidelines, the review addresses three objectives: (1) identifying the conceptual components of algebraic thinking, (2) examining recurring cognitive and pedagogical challenges in developing algebraic habits of mind, and (3) deriving implications for instructional design that cultivates structural reasoning and representational fluency. Findings indicate that difficulties often stem from fragmented representational understanding, limited structural awareness, and procedural teaching orientations. The review underscores the importance of integrating multiple representations, scaffolding generalization and justification, and aligning instruction with reasoning-centered practices. These insights inform the design of evidence-based professional learning programs aimed at supporting teachers in cultivating algebraic thinking and algebraic habits of mind.

Keywords: Algebraic thinking, Algebraic habits of mind, Systematic review

Introduction

Algebraic thinking is a cornerstone of mathematical literacy, serving as a bridge between arithmetic reasoning and generalized mathematical structures (Kaput, 1999; Kieran, 2007). Beyond procedural fluency, it enables learners to recognize patterns, express relationships, and generalize regularities through multiple forms of representation (Driscoll, 1999; van de Walle et al., 2016). However, research consistently shows that many students encounter persistent challenges in transitioning from arithmetic to algebra, particularly in grasping variables, equivalence, and structural relationships (Booth, 1984; Falkner et al., 1999). These challenges indicate that the development of algebraic thinking requires not only conceptual understanding but also a transformation in learners' ways of reasoning about mathematical relationships.

Driscoll (1999) conceptualizes this transformation in terms of algebraic habits of mind—cognitive dispositions that guide how individuals interpret, connect, and operate on mathematical structures. These habits—doing and undoing, building rules to represent functions, and abstracting from computation—equip learners to navigate flexibly between procedural, structural, and functional perspectives (Driscoll et al., 2001). Yet, existing research suggests that such habits are seldom foregrounded in classroom instruction, where learning environments often remain procedural and symbol-driven (Carraher & Schliemann, 2019; Stacey, 2015). Consequently, students' reasoning frequently remains context-bound, and their representational fluency and generalization skills develop in fragmented ways.

Addressing these issues calls for a systematic understanding of the conceptual and cognitive demands of algebraic thinking, as well as the instructional conditions that support its development. Among various frameworks proposed in the literature, Driscoll's model of algebraic habits of mind stands out for its focus on nurturing reasoning dispositions through structured learning experiences rather than isolated content delivery (Driscoll, 1999). This perspective aligns with contemporary views of algebraic thinking as a way of reasoning—a capacity to identify and operate on structural relationships—rather than a set of symbolic skills (Blanton et al., 2015; Kaput, 1999).

Given the central role of teachers in cultivating such reasoning-oriented learning environments, synthesizing empirical insights on how algebraic thinking and habits of mind emerge and evolve becomes a critical step in informing instructional design. Systematic reviews serve this purpose by integrating findings on developmental trajectories, recurring learning challenges, and effective pedagogical approaches, thereby building a foundation for designing evidence-informed professional learning opportunities.

Accordingly, this study presents a systematic review conducted within the scope of a TÜBİTAK (The Scientific and Technological Research Council of Turkey) 1001 Project (No. 223K317), which seeks to inform the design of teacher professional development initiatives aimed at fostering algebraic thinking and the associated habits of mind. Guided by the PRISMA 2020 protocol (Page et al., 2021), the review synthesizes research published between 2009 and 2024 to address the following aims:

- Identify the key conceptual components of algebraic thinking,
- Examine the recurring cognitive and pedagogical challenges in developing algebraic habits of mind, and
- Derive implications for instructional approaches that systematically cultivate these habits.

By mapping the research landscape through these lenses, the review offers an evidence base to guide the design of learning experiences that promote structural reasoning, generalization, and representational fluency—core dimensions of algebraic thinking. The next section reviews theoretical and empirical studies on the nature, developmental trajectories, and core components of algebraic thinking, alongside frameworks that have informed its instructional design. This is followed by a description of the PRISMA-based methodology employed in the review, after which the paper presents key findings and discusses their implications for teacher learning and instructional design.

Literature Review

Algebraic thinking has long been recognized as a cornerstone of mathematical reasoning, encompassing the capacity to identify patterns, discern structures, generalize relationships, and employ multiple representations to make sense of quantitative situations (Kaput, 1999; Driscoll, 1999; van de Walle et al., 2016). Rather than a discrete domain of symbolic manipulation, it represents a way of reasoning that organizes mathematical ideas through abstraction and generalization. In this sense, algebraic thinking contributes to the broader goal of developing learners' capacity for structural awareness and relational understanding, forming a bridge between arithmetic reasoning and more advanced forms of mathematical abstraction. Yet, numerous studies have revealed persistent challenges in this developmental transition, particularly when learners are required to reason beyond procedural computation and engage with symbolic representations as generalized expressions of relationships (Kieran, 2007). Misconceptions about variables, operations, and equivalence often limit students' ability to treat algebraic expressions as dynamic structures rather than static formulas. Such findings underscore that cultivating algebraic thinking is not merely a matter of introducing new content, but involves reorganizing cognitive processes around new forms of mathematical reasoning (Driscoll, 1999).

The development of algebraic thinking unfolds gradually through identifiable stages, as learners' attention shifts from numerical computation toward structural and functional reasoning. Hart et al. (1998) delineated a four-level model tracing this progression: from an initial reliance on arithmetic reasoning, through partial procedural use of symbols, to an emergent conception of letters as variables, and finally, to an integrated understanding that unites procedural fluency with structural insight. This trajectory suggests that algebraic competence depends on coordinating multiple ways of knowing—procedural, conceptual, and representational—rather than mastering symbolic techniques in isolation. Systematic research into these developmental transitions provides a valuable empirical foundation for identifying the conceptual demands learners encounter as they move toward generalized forms of reasoning.

Complementary theoretical accounts have articulated the core components of algebraic thinking from different vantage points. Kaput (1999) described it as a multi-dimensional construct organized around five interrelated themes: generalizing arithmetic patterns, using symbols meaningfully, recognizing structures within number systems, linking patterns to functional relationships, and engaging in mathematical modeling. These dimensions collectively frame algebraic thinking as a process of making sense of quantitative situations through recognition, representation, and generalization of structure. Similarly, Chimoni et al. (2018) proposed four broad dimensions—generalized arithmetic, functional thinking, modeling, and understanding of core concepts such as equality, variable, and covariation—emphasizing that algebraic reasoning integrates symbolic fluency with conceptual coherence. Across these frameworks, the literature converges on the view that algebraic thinking involves seeing mathematical relationships as objects of thought, coordinating multiple representational forms, and grounding generalizations in structural understanding.

Within this conceptual landscape, Driscoll's (1999, 2001) notion of algebraic habits of mind offers a process-oriented lens for examining how learners engage with algebraic ideas. Rather than focusing solely on the outcomes of instruction, this framework emphasizes the habitual ways of thinking that underlie algebraic reasoning. Driscoll identifies three interconnected habits: doing and undoing, building rules to represent functions, and abstracting from computation. The first involves reasoning bidirectionally, constructing and reversing mathematical processes, and understanding operations as invertible relationships. The second centers on detecting regularities, articulating them as generalized rules, and representing them across verbal, tabular, graphical, and symbolic forms. The third highlights the ability to move beyond specific numerical calculations to attend to the invariant relationships that structure them. Together, these habits describe how learners build, test, and refine algebraic generalizations through reflective engagement with mathematical structure. The framework thus extends prior component-based accounts by emphasizing not only what learners know about algebra, but how they think algebraically.

Despite extensive research on these constructs, recurring difficulties have been documented across studies. Learners often focus on surface features of patterns rather than functional relationships, leading to rules with limited generality or explanatory power (Blanton et al., 2015). Transitions among different representations—verbal, tabular, graphical, symbolic—pose further challenges, constraining students' ability to coordinate multiple perspectives (Nathan & Koedinger, 2000; Ayalon & Even, 2015). The habit of doing and undoing is likewise difficult to cultivate; many learners treat operations as unidirectional procedures, showing limited awareness of inverse reasoning or bidirectional relationships (Booth, 1984; Kieran, 2007). Similarly, in the realm of abstracting from computation, students often remain tied to context-specific procedures and fail to recognize underlying regularities, thereby missing opportunities for generalization and justification (McNeil & Alibali, 2005; Star et al., 2008; Wilkie, 2016). Collectively, these findings suggest that challenges in developing algebraic reasoning arise less from isolated misconceptions than from the absence of flexible, structure-oriented habits of mind.

Synthesizing these lines of research through a systematic review provides a dual contribution. First, it clarifies the conceptual terrain of algebraic thinking by mapping how its components, developmental stages, and cognitive habits have been theorized and empirically examined. Second, it identifies recurrent difficulties and pedagogical tensions that inform the design of learning experiences intended to foster deeper structural reasoning. Rather than prescribing specific interventions, such a synthesis yields an analytic framework for interpreting learners' algebraic engagement and for recognizing the cognitive shifts necessary to support their progression. In this way, the review not only consolidates theoretical and empirical insights but also illuminates how these insights can guide future research and instructional design aimed at nurturing algebraic thinking as a central dimension of mathematical understanding.

Method

The present study adopted a systematic literature review design guided by the PRISMA 2020 framework (Page et al., 2021) to synthesize empirical and theoretical research on algebraic thinking and the development of algebraic habits of mind. The review aimed not to prescribe a particular instructional design, but to generate a structured understanding of how prior scholarship has conceptualized, fostered, and assessed the development of algebraic reasoning across learning contexts. In this sense, the study sought to identify conceptual anchors, developmental challenges, and recurring reasoning structures that may inform future design-oriented research.

The data identification process was conducted through the Consensus academic search engine, a platform that integrates results from multiple high-impact databases including Google Scholar, ERIC, SpringerLink, and Elsevier. This platform was selected for its ability to deliver comprehensive, current, and filtered access to peer-reviewed publications while leveraging machine learning–based ranking algorithms that prioritize high-quality, indexed research (Consensus, 2024). This ensured a broader coverage of the interdisciplinary literature on algebraic thinking, which spans mathematics education, cognitive development, and curriculum design.

Search queries were developed iteratively through a combination of deductive and inductive refinement. Core search terms were derived from seminal frameworks (e.g., Kaput, 1999; Driscoll, 1999) and included algebraic thinking, early algebra, symbol sense, generalization in mathematics, functions and covariation, pattern generalization, multiple representations in algebra, and algebraic habits of mind. Boolean operators and truncations were used to expand and refine the search. The initial search yielded a broad corpus of publications, from which duplicates and irrelevant results were removed through systematic screening.

Following PRISMA's procedural structure, the screening and eligibility assessment were conducted in multiple stages. In the first stage, all retrieved studies were screened at the title and abstract level to exclude papers not directly addressing algebraic thinking or related constructs. In the second stage, full-text analyses were performed to assess the conceptual and methodological alignment of each study with the review's aims. The inclusion and exclusion criteria were operationalized as follows.

Studies were included if they explicitly investigated algebraic thinking, algebraic reasoning, or algebraic habits of mind as central constructs; were conducted in K–12 or teacher education contexts; reported empirical findings or proposed theoretically grounded frameworks; were published between 2009 and 2024; and appeared in peer-reviewed journals, edited books, or systematic reviews. Studies were excluded if they focused exclusively on university-level mathematics, examined technological tools without pedagogical analysis, addressed topics tangentially related to algebraic reasoning, or lacked accessible full texts. Each study was independently reviewed by two researchers to ensure coding reliability and conceptual coherence. Disagreements were resolved through consensus discussion, and a third reviewer was consulted in cases of persistent divergence. This process enhanced the trustworthiness of inclusion decisions and ensured consistency in the interpretation of conceptual relevance.

At the conclusion of the selection process, 41 studies were retained for synthesis. These publications collectively span a wide spectrum of perspectives, including historical analyses of algebra, theoretical frameworks of algebraic reasoning, studies on transitions from arithmetic to algebra, investigations into number theory and early algebraic structures, research on generalization and justification, examinations of structural reasoning, explorations of symbol sense, and analyses of representational fluency. The dataset thus reflects the multidimensionality of algebraic thinking as documented in the field.

The data extraction process followed a structured protocol capturing four dimensions: (1) bibliographic and contextual information (authors, year, educational level), (2) conceptual focus (e.g., generalization, functional reasoning, structure sense), (3) analytical orientation (empirical, theoretical, or design-based), and (4) key findings and implications. Extracted data were synthesized using a hybrid thematic approach (Thomas & Harden, 2008), combining deductive mapping to existing theoretical models (e.g., Kaput, 1999; Chimoni et al., 2018; Driscoll, 2001) with inductive identification of emergent patterns across the dataset. Through iterative cycles of coding, comparison, and abstraction, recurrent conceptual strands were identified that illuminate the progression, challenges, and cognitive dimensions of algebraic thinking.The overall review process—from identification to inclusion—is summarized in Table 1. The table illustrates the number of records at each stage and the filtering logic applied to arrive at the final dataset.

Table 1. Summary of systematic review process (PRISMA 2020)

Phase	Description	Number of Records
Identification	Records retrieved through Consensus search engine (2009–2024)	314
Screening	Records screened at title and abstract level	176
Eligibility	Full-text articles assessed for inclusion	91
Inclusion	Studies meeting all inclusion criteria	41

The final dataset, composed of 41 studies, provides a robust empirical and theoretical foundation for synthesizing the conceptual dimensions, developmental trajectories, and habitual reasoning processes central to algebraic thinking. The synthesis of these studies informs a nuanced understanding of how learners and teachers construct, interpret, and generalize mathematical relationships, offering a theoretically grounded lens for subsequent analysis in the following sections.

Findings and Discussion

This systematic review, based on 41 peer-reviewed studies published between 2006 and 2024, provides a comprehensive synthesis of the theoretical and empirical landscape surrounding algebraic thinking and the development of algebraic habits of mind. The thematic synthesis revealed four major strands: (1) historical and conceptual foundations, (2) developmental pathways and transitions from arithmetic to algebra, (3) core components and reasoning processes, and (4) cognitive and pedagogical challenges. Together, these themes illuminate the multifaceted nature of algebraic reasoning and highlight the structural, cognitive, and pedagogical factors that inform its development.

Historical and Conceptual Foundations of Algebraic Thinking

A recurring emphasis across the literature is that algebraic thinking cannot be reduced to procedural competence or symbolic manipulation; rather, it represents a distinct cognitive orientation toward structure, generality, and relational reasoning (Kaput, 1999; Driscoll, 1999; van de Walle et al., 2016). Studies examining the historical evolution of algebra underscore how the epistemological roots of algebra—as a language of relationships rather than operations—provide meaningful entry points for learners. Di Sia (2016), Maggio (2020), and Katz and Barton (2007) demonstrated that integrating historical narratives into instruction helps students situate algebraic ideas within broader intellectual traditions, promoting conceptual depth and reflective understanding. The inclusion of historical perspectives was consistently linked to improved metacognitive awareness and conceptual coherence: learners were more likely to interpret algebraic expressions as representations of relationships, not just as procedural tasks. This suggests that contextualizing algebra within its historical evolution can cultivate a more relational and structural view of mathematical knowledge—a key premise in fostering algebraic habits of mind.

Developmental Pathways and Transitions from Arithmetic to Algebra

A second major finding concerns the developmental progression from arithmetic reasoning toward algebraic reasoning. Numerous studies (Pratiwi et al., 2019; Hidayanto & Lathifa, 2021; Torres et al., 2023) identified persistent cognitive discontinuities in this transition. Students often struggle to detach from arithmetic-bound proceduralism, treating symbols as concrete objects rather than as placeholders for generalized quantities. This difficulty reflects a broader challenge of structural abstraction—a shift from computation to representation.

Evidence across the corpus highlights that relational thinking plays a bridging role in this transition (Andini & Prabawanto, 2021; Dekker & Dolk, 2011). When instruction foregrounds pattern recognition, equivalence, and relationships among operations, learners demonstrate greater facility in moving from specific instances to generalized forms. Studies on generalized arithmetic (Kaput et al., 2008; Pittalis et al., 2018) further reveal that fostering number sense and understanding factorization, divisibility, and numerical structure enhances readiness for formal algebraic reasoning. This developmental lens underscores the need for learning trajectories that scaffold abstraction incrementally—embedding symbolic reasoning within familiar numerical contexts before transitioning to formal algebraic structures. Such continuity supports the internalization of algebraic habits, particularly the ability to view operations as reversible and to reason flexibly about unknowns.

Core Components and Reasoning Processes in Algebraic Thinking

Generalization and Justification

Across the reviewed studies, generalization emerged as a foundational process in algebraic reasoning. Research demonstrates that learners' capacity to identify regularities, articulate general forms, and justify their validity underpins deep algebraic understanding (Ayala-Altamirano & Molina, 2021; Ellis, 2007). Yet, many learners generate incomplete or unsubstantiated rules, indicating limited understanding of the logical structure underlying generalizations. Studies by Pinto and Cañadas (2019) and Warren and Cooper (2006) showed that explicit justification practices—such as classroom discourse and representational reasoning—help transform intuitive observations into formalized generalities.

These findings affirm that generalization and justification are intertwined: the act of explaining why a pattern holds across cases reflects an internalized sense of mathematical necessity. Hence, algebraic thinking should be nurtured through iterative cycles of noticing, conjecturing, and validating relationships—a process directly aligned with habits of making and reversing processes and building functional rules.

Symbol Sense

A second core component, symbol sense, refers to learners' ability to interpret, manipulate, and assign meaning to symbols in context (Christou & Vosniadou, 2006; Rini et al., 2021). Many studies documented persistent misconceptions, such as interpreting letters as labels or specific numbers rather than as variables. Tasks emphasizing semantic interpretation of symbols—rather than mechanical manipulation—were shown to enhance structural understanding (Jupri & Sispiyati, 2021).

Findings also indicate that symbol sense develops through reflective engagement with multiple representations, where students connect symbolic expressions to graphical and verbal forms (Mielicki, 2015). Thus, cultivating symbol sense requires pedagogies that position symbols as meaning-bearing constructs, not procedural placeholders—an essential condition for developing algebraic habits grounded in reasoning and structure.

Structure Sense and Representational Fluency

Research focusing on structure sense highlights learners' ability to perceive algebraic expressions as structured wholes rather than linear sequences of operations (Hoch & Dreyfus, 2005; Kinach, 2014; Junarti et al., 2022). This capacity enables recognition of equivalence, manipulation of expressions, and flexible problem-solving. Visual and structural scaffolds (Apsari et al., 2015) were found to support recognition of invariant relationships, while representational fluency—moving among verbal, symbolic, graphical, and tabular forms—emerged as a critical predictor of conceptual integration (Panasuk, 2011; Erbilgin & Gningue, 2023).

The reviewed evidence underscores that representational transitions serve as cognitive bridges linking intuitive and formal reasoning. However, students often struggle to coordinate these forms, reinforcing the need for instruction that explicitly supports cross-representational mapping and meta-representational competence (Knuth, 2000).

Abstracting from Computation

Finally, studies addressing abstraction from computation (Driscoll, 2001; Carpenter et al., 2003; Wilkie, 2016) reveal that learners frequently engage in calculation without recognizing underlying structures. Encouraging abstraction—by reasoning about operations independent of specific numbers—fosters a shift toward structural generalization. Learners who can articulate equivalences, anticipate outcomes without full computation, and justify shortcuts demonstrate higher-order reasoning indicative of algebraic habits of mind. These findings reinforce that algebraic proficiency depends not only on procedural fluency but on the ability to extract and manipulate generalized relationships.

Cognitive and Pedagogical Challenges

Despite significant advances in conceptualizing algebraic thinking, the reviewed studies reveal systemic challenges that constrain learners' development of flexible, relational reasoning. Chief among these is the dominance of procedural instruction, which privileges rote manipulation over structural understanding (McNeil & Alibali, 2005; Star et al., 2008). Learners habituated to single-directional procedures often fail to engage in reversible reasoning, a cornerstone of algebraic thought (Booth, 1984; Hoch & Dreyfus, 2005). Similarly, the compartmentalization of representations within curricula leads to fragmented understanding, limiting students' ability to transition across symbolic, graphical, and verbal domains (Nathan & Koedinger, 2000; Warren et al., 2006).

Moreover, persistent misconceptions about equality and variable use indicate deep-rooted epistemological barriers, not merely content gaps. Addressing these requires instructional approaches that foreground conceptual coherence, structural reasoning, and discursive justification, thereby fostering cognitive habits aligned with algebraic thinking. Pedagogical designs that integrate pattern-based inquiry, historical contextualization, and multi-representational exploration appear particularly effective in overcoming these obstacles.

Synthesis and Implications

Synthesizing across these themes, the review reveals that the development of algebraic thinking involves a progressive reorganization of cognitive structures—from concrete calculation toward abstract, relational, and reversible reasoning. The cultivation of algebraic habits of mind depends on learners' sustained engagement with tasks that require recognizing invariance, constructing and testing generalizations, reasoning bidirectionally, and interpreting symbols meaningfully across contexts.

While the reviewed studies vary in scope and methodology, collectively they converge on a core insight: algebraic reasoning is not acquired through procedural practice but through epistemic apprenticeship into structural thinking. Future research and instructional design efforts should thus focus on creating learning environments that scaffold abstraction, valorize reasoning, and position learners as active generalizers and justifiers within rich representational landscapes. By illuminating the conceptual terrain and recurring challenges documented across 41 studies, this synthesis provides a research-informed foundation for advancing both theoretical understanding and pedagogical innovation in the teaching and learning of algebra.

Conclusion and Implications

This systematic review examined 41 peer-reviewed studies to identify how algebraic thinking and algebraic habits of mind have been defined, developed, and investigated across educational contexts. The synthesis indicates that algebraic reasoning emerges not merely from procedural mastery, but through sustained engagement with structural, relational, and reflective reasoning practices. Collectively, the studies point to algebraic thinking as a gradual cognitive transition—from performing operations to interpreting and constructing the relationships that underlie them. Supporting this transition requires instructional attention to generalization, justification, reversibility, abstraction, and representational connections.

The review brings together multiple conceptual perspectives—such as generalized arithmetic, functional thinking, structure sense, symbol sense, and abstraction from computation—to illustrate the multifaceted nature of algebraic reasoning. By tracing how these perspectives intersect and where learners encounter persistent challenges, the synthesis clarifies that developing algebraic habits of mind depends on structured opportunities for reasoning, pattern exploration, and meaning-making rather than isolated content exposure. In this sense, the review highlights shared insights rather than proposing a unified theory, emphasizing how different frameworks collectively inform understandings of algebraic cognition.

From a practical standpoint, the findings suggest the value of instructional designs that bridge arithmetic and algebraic reasoning through sequenced experiences and task variation. Approaches that integrate pattern-based exploration, multiple representations, and historical context appear particularly conducive to fostering flexible reasoning. Learning environments should invite students to construct and reverse processes, formulate rules, and identify structural regularities, encouraging reflection on why procedures work, not only how they are applied.

In conclusion, this review provides a research-informed overview of the conceptual and empirical foundations underlying algebraic thinking and algebraic habits of mind. Rather than offering prescriptive solutions, it serves as a reference point for researchers and educators seeking to design learning experiences that nurture deeper, more connected forms of algebraic reasoning. Future research might build on these insights to examine how design-based interventions can systematically support learners in thinking algebraically across developmental stages.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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