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Learning Obstacles in Constructing Proofs of Geometric Transformation: A Didactic Transposition Analysis

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Abstract: This study aims to identify and analyze the learning obstacles experienced by preservice mathematics teachers in constructing proofs of geometric transformations through a didactic transposition analysis from scholarly knowledge to learnt knowledge. A descriptive qualitative approach was employed, involving textbook analysis, classroom observations, diagnostic tests, and interviews with 19 preservice teachers. The findings indicate that learning obstacles are systemic and layered, emerging through the transformation of knowledge across transposition stages. These obstacles comprise epistemological, didactical, and ontogenetic dimensions that are interrelated. Epistemologically, obstacles arise from a reduction in deductive structure and conceptual formality, reflected in students' difficulties with quantifiers, proof logic, and the relationships among formal definitions of functions, injective functions, and surjective functions. Proof is often reduced to empirical generalization and visual verification, while formal representations such as two-variable functions and the domain $\mathbb{R}^2$ remain conceptually underdeveloped. Didactically, obstacles are reinforced by instructional practices characterized by dominant visual representations, limited proof strategies, and minimal opportunities for students to construct deductive arguments. Ontogenetically, students exhibit insufficient readiness for formal abstraction, shaped by predominantly procedural and illustrative learning experiences. These findings demonstrate that learning obstacles are not merely individual difficulties but are produced through mismatches across levels of knowledge in the didactic transposition process, highlighting the need for instructional designs that preserve epistemological coherence.

Keywords: Didactic transpositon, Geometric transformastion, Learning obstacles.

Introduction

Subdivide Proof is a fundamental element in mathematics as it serves as the basis for legitimizing every claim that is produced. In mathematics education, proof functions not only as a means of verification but also as a means of explaining conceptual relationships, constructing knowledge structures, and developing deductive ways of thinking (Hanna & de Villiers, 2012). The activity of proving enables students to interpret the meaning of definitions, trace the logical implications of assumptions, and integrate various mathematical ideas coherently (Varghese, 2017). Therefore, the ability to construct deductive proofs is an essential competence that must be possessed by preservice mathematics teachers.

In the study of geometric transformations, the demands of proof become increasingly complex due to the strong epistemological foundation underlying the concept of transformation. According to the Erlangen Program, geometry is understood as the study of properties that remain invariant under transformations (Katz, 2009). Within this framework, a transformation on the plane is formalized as a bijective function that maps a set of points onto itself (Martin, 2012). Consequently, understanding transformations cannot be limited to interpreting them as geometric motions, but must include the ability to establish their formal properties, including injectivity, surjectivity, and the preservation of geometric structures (Eccles, 1979; Umble & Han, 2014). Thus, proving geometric transformations requires students to construct deductive arguments that are consistent with these formal definitions.

However, numerous studies indicate that students still encounter difficulties in constructing formal proofs. One of the main factors contributing to this condition is the limited mastery of mathematical logical structures, such as the use of quantifiers, relationships between statements, and the role of definitions in proof (Epp, 2003). In the context of geometric transformations, these difficulties become more complex as students are required to integrate their understanding of functions, geometric representations, and deductive argumentation within a coherent framework. As a result, the proofs they produce are often unsystematic, incomplete, and do not yet meet the standards of mathematical rigor.

Previous studies have shown that research on proving geometric transformations has been largely dominated by efforts to identify students' difficulties, errors, and learning obstacles. These findings consistently reveal difficulties in understanding the definition of transformation as a function, errors in determining domain and codomain, confusion in distinguishing between injective and surjective properties, and limitations in visualizing mappings (Kusuma & Setyaningsih, 2015; Noto et al., 2019; Napfiah & Sulistyorini, 2021; Indahwati, 2023; Aulia et al., 2023; Maifa, 2019). More recent studies further indicate that preservice teachers predominantly remain at the stages of personalization and contextualization, experience conceptual, procedural, and visual errors, and rarely reach the stages of depersonalization and decontextualization in constructing proofs of transformations (Maifa, et al, 2025).

These findings not only demonstrate the existence of difficulties in proof but also indicate a more fundamental issue, namely students' inability to construct deductive arguments that are consistent with formal definitions. In many cases, proof is reduced to procedural activities or verification based on examples and visual representations. This condition aligns with the concept of empirical proof schemes, in which the truth of a statement is accepted based on limited observations (Hanna & de Villiers, 2012). Although such approaches may support the development of initial intuition, they are insufficient for achieving formal deductive proof. If this condition is not addressed, preservice teachers may develop mathematical understanding that is procedural and intuitive, and subsequently reproduce similar patterns of thinking in their future teaching practices.

Ideally, the teaching of geometric transformations should guide students to understand transformations as bijective functions established through deductive arguments grounded in formal definitions. Students are expected to connect definitions with proof structures, use mathematical logic consistently, and construct systematic justifications. However, the gap between this ideal condition and actual classroom practice suggests that students' difficulties cannot be viewed merely as individual problems, but are also closely related to how mathematical knowledge is transformed within educational contexts.

From the perspective of the Anthropological Theory of the Didactic (ATD), this transformation of knowledge can be analyzed through the concept of didactic transposition, which refers to the process of transforming knowledge from its scholarly form into knowledge to be taught, implemented in instruction, and eventually understood by students (Chevallard, 2019). Each stage in this process involves selection, interpretation, and adaptation that may alter the epistemological structure of the knowledge. To examine these transformations systematically, the praxeological framework is employed to analyze the relationships among types of tasks, solution techniques, and the justifications underlying them.

However, most previous studies have not examined learning obstacles in relation to the process of didactic transposition. In other words, students' difficulties have not been analyzed as outcomes of changes in knowledge structure along the trajectory from scholarly knowledge to learnt knowledge. Therefore, an approach is needed that can reveal how epistemological, didactical, and ontogenetic obstacles are systematically formed throughout this process.

Based on this rationale, this study aims to identify and analyze learning obstacles in proving geometric transformations through a didactic transposition analysis. By tracing the transformation of knowledge from scholarly knowledge to learnt knowledge, this study offers a more comprehensive perspective on the origins of learning obstacles. The novelty of this study lies in the integration of epistemological and didactical analyses to explain how learning obstacles are formed, thereby providing a foundation for designing instruction that can bridge the transition from intuitive understanding to formal deductive reasoning.

Method

Research Design

This study is a qualitative inquiry conducted within the framework of Didactical Design Research (DDR) (Suryadi, 2010), focusing on the diagnostic phase to identify learning obstacles in proving geometric transformations. The analysis was carried out using the framework of didactic transposition by tracing the transformation of knowledge from scholarly knowledge to knowledge to be taught, then to taught knowledge, and ultimately to learnt knowledge (Chevallard, 2019). To understand how knowledge is interpreted and transformed at each stage, this study integrates a phenomenological approach. Referring to Tuffour (2017), phenomenological analysis was conducted through four stages: bracketing, intuiting, analyzing, and describing, which were applied at each stage of the didactic transposition process.

Research Procedure

The research procedure followed three stages of didactic transposition as outlined below.

Scholarly Knowledge → Knowledge to be Taught

This stage examined how the formal structure of proving geometric transformations was transposed into textbooks. The analysis employed the praxeological framework (task, technique, technology, theory) to map the structure of the knowledge presented.

- Bracketing: The researchers established the formal proof structure as a reference and suspended immediate acceptance of the textbook presentation, then mapped the textbook content into praxeological components.
- Intuiting: The researchers closely examined how concepts, definitions, and proofs were represented in the textbooks.
- Analyzing: The praxeological structure was analyzed to identify alignment or reduction between scholarly knowledge and knowledge to be taught.
- Describing: The findings were presented in the form of narratives and praxeological tables that illustrate patterns of transposition and potential learning obstacles.

Knowledge to be Taught → Taught Knowledge

This stage investigated how the knowledge presented in textbooks was transformed into classroom instructional practice.

- Bracketing: The researchers suspended the assumption that the material was taught exactly as presented in the textbooks and focused on the lecturer's actual instructional practices.
- Intuiting: The researchers observed how the lecturer explained concepts, used representations, and presented proofs.
- Analyzing: The data were analyzed to identify shifts, simplifications, or particular emphases in the delivery of the material.
- Describing: The findings were presented in narrative form to describe the nature of taught knowledge and its implications for the emergence of learning obstacles.

Taught Knowledge → Learnt Knowledge

This stage analyzed how the knowledge delivered in instruction was understood and constructed by students.

- Bracketing: The researchers suspended assumptions regarding students' errors and focused on their actual responses.
- Intuiting: The researchers examined in depth how students understood definitions, constructed proofs, and used representations.
- Analyzing: Test and interview data were coded to identify recurring patterns of difficulties and errors.
- Describing: The findings were presented as descriptions of learnt knowledge patterns that indicate the emergence of learning obstacles.

Integration of Analysis

The results from the three stages of transposition were integrated to trace relationships across levels of knowledge. Learning obstacles were identified as consequences of mismatches between the structure of scholarly knowledge, the form of its presentation in instruction, and students' understanding. These obstacles were then categorized as epistemological, didactical, and ontogenetic (Brousseau, 2002).

Data Sources and Participants

The data sources encompassed four levels of didactic transposition. Data at the scholarly knowledge level were obtained from mathematical literature containing formal definitions of geometric transformations and their proof structures (Eccles, 1979; Martin, 1982), which served as the epistemological reference. Data at the knowledge to be taught level were obtained from textbooks used in the course and analyzed using the praxeological framework.

Data at the taught knowledge level were obtained from classroom observations and instructional materials used by the lecturer, whereas data at the learnt knowledge level were obtained from diagnostic tests, students' written work, and in-depth interviews. The participants consisted of 19 preservice mathematics teachers enrolled in a Geometric Transformation course, selected through purposive sampling.

Results and Discussion

Didactic Transposition from Scholarly Knowledge to Knowledge to be Taught: A Praxeological–Phenomenological Analysis

This stage aims to examine how the structure of proving geometric transformations is transposed from scholarly knowledge into knowledge to be taught as represented in the lecturer's textbook. From an epistemological perspective, geometric transformations are understood as bijective functions on the plane; therefore, their proofs encompass the proof of function, injectivity, and surjectivity as an integrated deductive structure.

The analysis was conducted by integrating the praxeological framework, comprising task (T), technique (τ), technology (θ), and theory (Θ), with the stages of phenomenological analysis, namely bracketing, intuiting, analyzing, and describing. This integration enables not only the identification of the knowledge structure but also an interpretation of the variations that emerge throughout the transposition process.

(1) Bracketing: Identification of the Praxeological Structure

At this stage, the structure of proof was identified and decomposed into praxeological components without involving interpretation. The analysis was grounded in scholarly knowledge derived from established mathematical references on geometric transformations, particularly those that define transformations as bijective functions and emphasize formal deductive structures (Eccles, 1979; Martin, 1982; Umble & Han, 2014).

The primary focus was to map how the proof of bijective functions, encompassing function, injectivity, and surjectivity, is represented across different sources, including scholarly references and textbooks. A summary of the identification results is presented in Table 4.1.

Table 4.1. Praxeology of proof in geometric transformations (Comparative)

Component	Scholarly Knowledge	Textbook A	Textbook B
Task (T)	Proving that a mapping is a bijective function	Determining transformations through examples	Determining transformations through stepwise proof
Technique (τ)	Various approaches: direct proof, contradiction, constructive methods, inverse	Sketches, diagrams, and limited examples	Deductive reasoning based on cases
Technology (θ)	Formal definitions based on quantifiers and inverse	Definitions are presented but not operationalized	Definitions are implicitly used as a basis for proof
Theory (Θ)	Function theory, logic, and Euclidean geometry	Not integrated into practice	Partially integrated into proof

(2) Intuiting: Initial Meaning-Making

Based on Table 4.1, initial differences can be observed in how proofs are constructed across the three knowledge levels. In scholarly knowledge, proofs are structured deductively with a variety of valid strategies. In contrast, Textbook A demonstrates a tendency to rely on illustrative representations grounded in examples, whereas Textbook B moves toward a more systematic deductive structure.

(3) Analyzing: Analysis of the Praxeological Structure

Further analysis reveals that these differences occur across several levels of praxeology.

At the level of technique (τ), scholarly knowledge provides a range of mathematically valid proof strategies. Textbook A is dominated by the use of examples, sketches, and diagrams as the basis for justification. Meanwhile, Textbook B adopts a deductive approach based on case analysis, although with limited variation in strategies. At the level of technology (θ), formal definitions in scholarly knowledge function as an explicit foundation for each step of the proof. In Textbook A, these definitions are not employed as the basis of argumentation but are merely presented. In Textbook B, definitions are utilized within the proof process, although they are not always articulated explicitly as part of a coherent conceptual structure.

In terms of the relationship between praxis and logos, scholarly knowledge exhibits a strong integration between techniques and their theoretical justifications. In contrast, Textbook A tends to emphasize practice without formal justification, while Textbook B shows a clearer connection between technique and justification, albeit without encompassing the full range of epistemologically available strategies.

(4) Describing: Description of the Transposition Phenomenon

Based on this analysis, the process of transposing scholarly knowledge into knowledge to be taught results in variations in the organization of proofs of geometric transformations. On the one hand, proof is represented through an illustrative approach that emphasizes examples and visualizations. On the other hand, proof is structured deductively but with a relatively limited set of strategies. In contrast, at the level of scholarly knowledge, proof is constructed deductively with a broader range of strategies and an explicit connection between formal definitions and the steps of proof.

These findings indicate that there are variations in the relationship between formal definitions, proof strategies, and the implementation of techniques in the presentation of geometric transformation proofs at the level of knowledge to be taught.

Didactic Transposition from Knowledge to be Taught to Taught Knowledge: A Phenomenological Analysis

This stage aims to analyze how knowledge to be taught is transformed into taught knowledge through classroom instructional practices. The analysis focuses on how the lecturer selects, organizes, and presents the material on proving geometric transformations, including the use of instructional materials and the strategies employed during instruction. The data were obtained through classroom observations, allowing the instructional process to be traced chronologically and enabling the identification of transformations in the structure of knowledge during its implementation. The analysis was conducted using the stages of phenomenological inquiry, namely bracketing, intuiting, analyzing, and describing.

(1) Bracketing: Segmentation of Instructional Practice

At this stage, classroom instructional practices were identified and segmented into units of phenomena without interpretation. This segmentation aims to clarify the structure of how proof-related material is presented in actual classroom settings. The results of this segmentation are presented in Table 4.2.

Table 4.2. Segmentation of students' response patterns in proving transformations

Code	Learning Segment	Description of Practice	Dominant Representation
B1	Presentation of definition	The lecturer presents the definition of transformation as a bijective function	Verbal + symbolic
B2	Review of prerequisites	Function, injective, and surjective concepts are explained using arrow diagrams	Visual
B3	Definition of injectivity	The definition of injectivity is written using formal notation	Symbolic
B4	Explanation of surjectivity	The concept of surjectivity is explained verbally without formal structure	Verbal
B5	Representation of function	The form $f((x,y))$ is introduced without conceptual elaboration	Symbolic
B6	Proof of injectivity	Proof is conducted using contradiction	Visual + symbolic
B7	Proof of surjectivity	Proof is conducted using inverse	Symbolic
B8	Use of sketches	Geometric sketches are used dominantly in explanations	Visual
B9	Student activity	Students follow the explanation without initial exploration	Passive

(2) Intuiting: Initial Interpretation of Instructional Patterns

Based on the segmentation presented in Table 4.2, it can be observed that the instructional process emphasizes the use of visual representations and predetermined procedures. Arrow diagrams and geometric sketches serve as the primary tools for explaining concepts, particularly in segments B2 and B8. Meanwhile, formal representations appear only in specific parts, such as in the definition of injectivity (B3), but are not consistently employed across other concepts, such as surjectivity (B4).

In addition, the proof strategies used in segments B6 and B7 exhibit a fixed pattern, where each property is established through a specific approach. The instructional process also demonstrates a lecturer-centered tendency, as reflected in segment B9, where students predominantly follow a pre-structured sequence of explanations.

(3) Analyzing: Analysis of the Transposition Structure

Further analysis reveals that transformations occur in how knowledge is represented and organized as it moves from knowledge to be taught to taught knowledge.

In terms of representation, segments B2 and B8 indicate a dominance of visual representations in explaining both functions and proofs. In contrast, segments B3 and B4 reveal differences in the level of formality across concepts. This suggests that formal representations are not consistently integrated throughout the instructional structure. In terms of proof strategies, segments B6 and B7 show that proofs are presented through relatively fixed strategic patterns. The range of proof strategies that are epistemologically available is not fully enacted in classroom practice. In terms of instructional structure, the connection between textbooks and classroom practice is evident through the similarity of task types employed. This indicates that the structure of knowledge to be taught directly influences the form of taught knowledge presented in the classroom. Furthermore, segment B9 indicates that students' roles in the learning process are largely limited to following the lecturer's explanations, resulting in limited opportunities to construct arguments independently.

(4) Describing: Description of the Taught Knowledge Phenomenon

Based on this analysis, taught knowledge in the teaching of proofs of geometric transformations is characterized by the dominance of visual representations, the use of structured yet limited proof strategies, and a lecturer-centered mode of instruction. The structure of proof that emerges in classroom practice tends to follow predetermined patterns, closely aligned with the instructional materials used. Meanwhile, the use of formal definitions and the development of deductive arguments do not consistently appear explicitly at every stage of instruction. This phenomenon indicates that, within the process of didactic transposition, adjustments occur in how concepts and proofs are represented, both in terms of modes of presentation, the strategies employed, and the extent of student engagement in constructing understanding.

Didactic Transposition from Taught Knowledge to Learnt Knowledge: A Phenomenological Analysis

This stage aims to trace how taught knowledge presented in instruction is internalized and reconstructed by students into learnt knowledge. The analysis focuses on students' actual understanding in representing transformation concepts and in constructing deductive proofs of function properties. The data were obtained through diagnostic tests administered to 19 students, complemented by in-depth interviews to further explore the reasoning underlying their responses. The analysis was conducted using the stages of phenomenological inquiry, namely bracketing, intuiting, analyzing, and describing.

(1) Bracketing: Segmentation of Students' Response Patterns

At this stage, students' responses were identified and segmented into patterns of errors and difficulties without interpretation. This segmentation aims to clarify the actual forms of knowledge constructed by students following instruction.

Table 4.3. Segmentation of students' response patterns in proving transformations

Code	Students' Response Pattern	Brief Description	Dominant Representation	Form of Reasoning
L1	Proof based on point examples	Using several points to conclude function properties	Visual + numeric	Empirical
L2	Image-based proof	Using sketches to directly conclude function properties	Visual	Illustrative
L3	Transformation = function	Failing to distinguish between functions and bijective transformations	Verbal	Partial
L4	Visualization error	Drawings do not conform to mapping rules	Visual	Inconsistent
L5	Diagram as proof	Arrow diagrams are considered as formal proof	Visual	Representational
L6	Injectivity error	Unable to construct a logical argument for injectivity	Partially symbolic	Procedural
L7	Surjectivity error	Unable to understand the preimage–image relationship formally	Visual+Verbal	Descriptive

(2) Intuiting: Initial Interpretation of Students' Response Patterns

Based on Table 4.3, it can be observed that most students rely heavily on visual representations and concrete examples in constructing their responses. Patterns L1 and L2 indicate a tendency to use specific points and sketches as the primary basis for concluding function properties. In addition, pattern L3 reveals that students do not consistently distinguish between the concepts of function and bijective transformation. In patterns L6 and L7, the proofs of injectivity and surjectivity tend to follow procedural steps without being supported by complete deductive arguments.

(3) Analyzing: Analysis of the Learnt Knowledge Structure

Further analysis indicates that the transformations occurring at the level of learnt knowledge are closely related to the forms of taught knowledge previously identified.

In terms of representation, the dominance of visual representations in classroom instruction (segments B2 and B8) is reflected in students' responses, particularly in patterns L1, L2, and L5, where visual and example-based reasoning becomes the primary means of justification. This suggests that visual representations are not merely supportive tools but become substitutes for formal reasoning. In terms of conceptual structure, the inconsistency in the use of formal definitions during instruction (segments B3 and B4) is manifested in students' partial understanding, as seen in pattern L3. Students tend to conflate the notion of function with bijective transformation, indicating an incomplete conceptual differentiation. In terms of proof strategies, the use of fixed strategies in instruction (segments B6 and B7) is reflected in students' procedural approaches, particularly in pattern L6. Students attempt to follow known steps without constructing logically coherent arguments, indicating a shift from deductive reasoning to procedural reproduction. In terms of student participation, the limited opportunities for independent exploration during instruction (segment B9) are reflected in students' difficulties in constructing proofs independently. Their responses tend to be descriptive or fragmented rather than systematically deductive, as seen across several patterns.

(3) Describing: Description of the Learnt Knowledge Phenomenon

Based on this analysis, learnt knowledge in the context of proving geometric transformations is characterized by a reliance on visual and empirical reasoning, partial conceptual understanding, and procedural approaches to proof construction. The structure of proof constructed by students tends to be fragmented, relying on examples, representations, or memorized steps rather than on coherent deductive arguments grounded in formal definitions. The relationships among function, injectivity, and surjectivity are not fully integrated into a unified conceptual framework.

This phenomenon indicates that, within the process of didactic transposition, the transformation of knowledge results in a form of understanding in which formal structures are reduced, representations are overemphasized, and deductive reasoning is not fully developed. Consequently, students construct knowledge that is not yet aligned with the epistemological structure of mathematical proof.

Learning Obstacles in a Didactic Transposition Perspective

Based on the didactic transposition analysis conducted from scholarly knowledge to learnt knowledge, this study reveals that learning obstacles in proving geometric transformations do not emerge as isolated phenomena at the student level. Rather, they are systematically formed through transformations in the structure of knowledge at each stage of transposition. The synthesis of findings from Sections 4.1, 4.2, and 4.3 indicates that the observed obstacles fall into three main categories—epistemological, didactical, and ontogenetic obstacles—as classified by Brousseau (2002). These categories do not operate independently; instead, they are interrelated and evolve through the dynamic interplay between the structure of knowledge, its didactical implementation, and students' readiness to construct deductive proofs.

From an epistemological perspective, the findings indicate that the primary obstacle lies in the reduction of the deductive structure of proof. At the level of scholarly knowledge, geometric transformations are constructed as bijective functions, requiring an explicit relationship between formal definitions, the use of quantifiers, and mathematically valid proof strategies. However, during the transposition into knowledge to be taught and taught knowledge, this structure undergoes simplification. Formal definitions are not consistently mobilized as the foundation of argumentation, the range of proof strategies becomes restricted, and proof practices shift toward more illustrative forms. Consequently, at the level of learnt knowledge, students tend to interpret proof as verification based on examples, visualizations, or fixed procedures, rather than as a deductive process derived from formal definitions.

These findings are consistent with previous studies showing that students experience difficulties in understanding formal definitions, constructing deductive proofs, and distinguishing properties of functions such as injectivity and surjectivity (Kusuma & Setyaningsih, 2015; Noto et al., 2019; Napfiah & Sulistyorini, 2021; Aulia et al., 2023; Maifa, 2019). The patterns of student responses observed in this study, such as reliance on point examples, dependence on visual representations, and the assumption that diagrams constitute proof, also reinforce the notion of dominant empirical proof schemes as described by Hanna and de Villiers (2012). However, unlike prior studies that primarily identify the forms of students' difficulties, this study demonstrates that such patterns are the result of epistemological reduction occurring throughout the process of knowledge transformation.

From a didactical perspective, the study shows that learning obstacles are not only rooted in the content itself but also in how that content is represented and enacted in instruction. At the level of knowledge to be taught, one textbook emphasizes illustrative and example-based approaches, while another adopts a more deductive orientation, albeit still limited in its range of strategies. At the level of taught knowledge, these tendencies are reinforced through lecturer-centered instructional practices, the dominance of arrow diagrams and geometric sketches, and the use of fixed proof strategies, such as contradiction for injectivity and inverse for surjectivity. Under such conditions, students tend to reproduce given proof patterns rather than construct arguments independently.

This finding extends previous research, which has shown that instruction in transformation geometry and proof is often procedural, visual, and insufficient in fostering deep understanding (Noto et al., 2019; Sunariah & Mulyana, 2020; Uygun, 2020). However, this study goes further by interpreting these pedagogical limitations as didactical obstacles in the sense proposed by Brousseau, namely, obstacles generated by the way knowledge is transformed and taught. In other words, the study does not merely suggest that instruction is "less effective," but provides a deeper explanation of how choices of representation, proof strategies, and classroom interaction patterns create conditions that hinder the construction of deductive reasoning.

From an ontogenetic perspective, the study also indicates that students are not yet fully prepared to engage with the level of formal abstraction required in proving geometric transformations. Difficulties in understanding functions as mappings on $\mathbb{R}^2$, distinguishing images and preimages formally, and using implication structures and quantifiers suggest that students are still operating at intuitive, visual, or procedural levels of understanding. While these findings align with studies indicating that students often struggle to transition from concrete representations to formal reasoning in geometry and proof, the contribution of this study lies in reframing ontogenetic obstacles not merely as internal deficiencies.

Through the lens of didactic transposition, it becomes evident that such ontogenetic limitations develop within a learning environment that has already undergone epistemological reduction and didactical constraint. Thus, ontogenetic obstacles in this study are not simply a matter of students being "unprepared," but are also the result of learning experiences that fail to provide an adequate bridge from visual representations to formal deductive structures. This constitutes a key contribution of the study: ontogenetic obstacles are conceptualized not as isolated individual factors, but as integral components of a broader system of knowledge transposition.

These three types of obstacles ultimately reinforce one another. Epistemological reduction at the level of knowledge leads to the loss of formal proof structure. This reduction is then reproduced didactically in classroom practice through dominant visual representations, limited proof strategies, and lecturer-centered instruction. Under such conditions, students lack sufficient opportunities to develop deductive reasoning, thereby intensifying ontogenetic obstacles. Conversely, students' ontogenetic limitations further increase their reliance on the representations and procedures provided didactically. This reciprocal relationship demonstrates that learning obstacles in proving geometric transformations cannot be explained from a single perspective, but must be understood as the result of interactions among epistemological, didactical, and ontogenetic dimensions.

Accordingly, the main contribution of this study lies in the use of didactic transposition analysis to explain the mechanisms underlying the systemic formation of learning obstacles. Unlike previous studies that primarily position students' difficulties as outcomes at the level of learning results, this study shows that such obstacles are products of misalignment between the epistemological structure of proof at the level of scholarly knowledge, its representation in knowledge to be taught, its enactment in taught knowledge, and its internalization in learnt knowledge. This is where the novelty of the study resides: not only in identifying that students experience difficulties in proving geometric transformations, but in explaining why and how these difficulties are formed through the chain of didactic transposition.

This perspective implies that efforts to improve the teaching of geometric transformation proofs cannot be limited to addressing student-level difficulties alone. Improvement must involve re-establishing coherence between formal definitions, proof strategies, visual representations, and the design of instructional situations. Instruction should be designed such that visualizations function as epistemic tools rather than substitutes for argumentation; formal definitions are consistently mobilized in proof; and students are provided with opportunities to construct, test, and validate deductive arguments independently. Through such an approach, the teaching of geometric transformation proofs can shift from procedural and illustrative patterns toward formal and meaningful mathematical reasoning.

Conclusion

This study demonstrates that learning obstacles in proving geometric transformations are not partial phenomena at the student level but are systematically constructed through the process of didactic transposition from scholarly knowledge to learnt knowledge. The identified obstacles encompass epistemological, didactical, and ontogenetic dimensions that are interrelated and mutually reinforcing, where epistemologically there is a reduction in the deductive structure and conceptual formality of proof, didactically instruction tends to emphasize procedural approaches and visual representations, and ontogenetically students show limited readiness to engage with formal abstraction in proof construction. These obstacles emerge from misalignments across levels of knowledge within the didactic transposition process, ultimately hindering the development of meaningful deductive reasoning.

The findings confirm that students' difficulties in proof are not merely the result of individual limitations, but rather the consequence of knowledge transformations that do not fully preserve epistemological continuity, and the main contribution of this study lies in demonstrating how learning obstacles are systematically produced through the chain of didactic transposition, thereby offering a more comprehensive explanation of their origin in the context of geometric transformation proofs.

Recommendations

Based on these findings, several recommendations can be proposed, particularly in relation to instructional design and future research. Instruction should be designed to re-establish coherence between formal definitions, proof structures, and mathematical representations, ensuring that visualizations function as epistemic supports rather than substitutes for deductive reasoning, while also providing opportunities for students to actively construct, test, and validate their own proofs instead of merely reproducing predefined procedures.

In addition, teaching should incorporate a wider variety of proof strategies and explicitly emphasize the use of formal definitions, quantifiers, and logical structures to support students' transition from intuitive and visual reasoning toward formal deductive reasoning. Furthermore, future research is needed to examine the applicability of these findings across broader educational contexts and to develop didactical designs that directly target the identified learning obstacles through more systematic and intervention-based approaches.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

* Ethical approval is not required for this study.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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