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Exploring Pupils' Mathematical Understanding: The Role of Metacognitive-Discursive Activities in the Learning Process

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Abstract: This paper reports on a research project conducted in a secondary school on Sumba Island, Indonesia, aiming to enhance pupils' understanding of mathematics. To achieve this, the learning environment was designed to reconstruct certain properties of mathematical objects and their related operations, with a specific focus on equations. At the beginning of the learning sequence, pupils engaged with number puzzle stories that prompted them to represent the given situations in algebraic form and solve the resulting equations. Throughout the lessons, metacognitive and discursive activities were applied to foster reflection, reasoning, and communication. Technological tools supported the implementation of these principles: a document camera and projector were used so that pupils could display and discuss their solutions with their classmates accurately. Through this setting, pupils learned that they were individually responsible for presenting and justifying their mathematical arguments in front of the class. This technological mediation using a document camera, fostered an interactive discussion environment that promoted both individual accountability and collective reasoning. After the learning sequence, a written test was administered to assess pupils' understanding of the mathematical concepts addressed. The test items were aligned with the targeted mathematical competencies. Selected pupil solutions are presented and discussed as illustrative examples. The findings indicate that metacognitive-discursive activities effectively supported pupils in developing a deeper and more connected understanding of mathematics.

Keywords: Problem solving, Metacognitive-Discursive activities, Equation

Introduction

Understanding mathematics requires more than memorizing formulas or executing fixed procedures; it involves recognizing relationships among ideas, interpreting the meaning behind operations, and applying reasoning to solve problems. One central dimension of this broader notion is what Kilpatrick, Swafford, and Findell (2001) describe as *conceptual understanding*—the comprehension of mathematical concepts and the connections among them, which enables learners to integrate knowledge in a meaningful way. However, school mathematics is often taught in ways that encourage procedural imitation rather than genuine understanding. Freudenthal (1986) emphasized this problem, stating:

"*Mathematics – at least algebra – is too often presented as a system of working according to prefabricated and then imposed rules. There is everything to be said in favor of preventing this impression as far as the learner is concerned. […] It is a view of mathematics, yet it is no mathematics, and certainly it is no mathematics to start with as a learner" (p. 467).*

This critique underscores that mathematics should not be viewed as a ready-made system but as a human activity in which learners construct and reconstruct meaning. Understanding grows through reconstruction rather

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than reproduction (Freudenthal, 1983), meaning that learners build mathematical knowledge actively rather than simply imitating procedures. Recognizing this gap, several scholars advocate for emphasizing discourse and metacognition in mathematics instruction. Classroom discourse, understood as the dialogic exchanges where learners negotiate meanings and challenge ideas, is central to developing deeper understanding (Sfard, 2008; Michaels, O'Connor, & Resnick, 2008). Through structured discussions, students are invited to articulate, justify, and critically examine reasoning—core practices in building mathematical understanding. Beyond discourse, contemporary research stresses the need for metacognitive activity as a driver of understanding. Kaune et al. (2012) show that metacognitive and discursive activities develop simultaneously and are interrelated within specially designed mathematics learning environments. These activities engage students in monitoring and reflecting not only on their own thinking processes but also on the thinking of others, which occurs through interactions among peers. The approach highlights the importance of both individual reflection and social interaction as integral components of mathematical understanding. Building on this idea, Kaune and Fresenborg (2017) designed a learning environment that not only reconstructs properties of mathematical objects and related operations but also helps students reconstruct mathematical ways of thinking connected to the axiomatic foundations and extensions of the number system. Research shows that learning environments designed to encourage metacognitive and discursive activities enable pupils to better understand mathematical concepts and procedures, rather than merely memorizing computational techniques (Kaune et al., 2012; Ate, et al., 2025). Such environments not only foster knowledge integration but also empower learners to actively engage with mathematical ideas, reflecting, discussing, and constructing meaning through continuous interaction.

To describe pupils' understanding in a more systematic and operational way, the present study refers to the framework of mathematical competencies developed by Blum, et al. (2006). This framework captures how pupils' mathematical understanding becomes visible through six interrelated competencies: arguing, problem-solving, modelling, representing, symbolic-formal reasoning, and communication. Accordingly, this study examines pupils' test solutions after participating in classroom practices that involved metacognitive-discursive activities to identify how these competencies are manifested in their mathematical work.

Method

This study employed a qualitative-descriptive design based on written test data. The purpose was to identify how pupils' mathematical competencies, as defined by Blum, et al. (2006), are manifested in their problem-solving work after participating in lessons that involved metacognitive-discursive activities. The participants were pupils from Grade 8 who had taken part in classroom practices designed according to the principles of metacognitive and discursive mathematics instruction (Kaune & Fresenborg, 2021). The teaching content in this study focused on equations. To introduce the topic, pupils worked on number-puzzle problems designed to encourage them to translate verbal information into algebraic representations and corresponding equations. Once the equations were constructed, they were required to apply appropriate algorithms to obtain the solutions. During these lessons, pupils were encouraged to explain their reasoning, discuss alternative approaches, and reflect on their problem-solving processes. Technological tools supported the implementation of these principles: a document camera and projector were used so that pupils could display and discuss their solutions with their classmates accurately. Through this setting, pupils learned that they were individually responsible for presenting and justifying their mathematical arguments in front of the class. This technological mediation fostered an interactive discussion environment that promoted both individual accountability and collective reasoning. After the learning sequence, a written test was administered to capture the mathematical competencies developed in this context. The test consists of three exercises. One of the exercises will be analyzed in this article. Solve the following problem.

The sum of three successive natural numbers is 90.

a. Write the mathematical expression or equation that represents the problem.
b. Show the steps of your solution process.
c. State the final answers:

First number : _______
Second number : _______
Third number : ______

This exercise represents a typical word problem designed to create opportunities for pupils to engage meaningfully with mathematics. With such tasks, it is very clear that there is one solution, that the data needed for the solution is available. The tasks are constructed in such a way that calculations work, even if the facts are not fully understood in mind (Kirsch, 1991). The first question aims to assess the students' abilities for making a formalization of a given story. The second question aims to solving an equation. Furthermore, it is a great interest whether and how far the students use the models offered in the textbook during they solve the problem. Besides that, the test assessed their competence in applying the contractual of arithmetic and using the equations theorem. The third question asks the students to decide what is the first number, second number, and third number.

Results and Discussion

Theory Based Analysis of the Task

In this study, we applied the framework of Blum et al. (2006) to evaluate students’ mathematical competencies when solving a given problem. Each task was categorized according to the specific competency it assesses:

Task a) corresponds to competence three (K3), mathematical modelling. Students are required to translate the information provided into an algebraic form. They must understand the problem context and construct a suitable mathematical model. This task is considered challenging because it demands the ability to convert textual information into precise mathematical formulas (Cohors-Fresenborg et al., 2004).
Task b) corresponds to competence five (K5), symbolic-formal reasoning. While solving the algebraic expression, students engage with variables, terms, equations, and functions, applying formal mathematical operations correctly.
Task c) corresponds to competence one (K1), mathematical argumentation. Students must construct a logical argument to determine the three consecutive numbers and verify that their sum equals 93. This task requires reasoning skills to justify each step and to support the solution obtained with valid mathematical arguments.

Students Solution

The students show competencies that enable them to connect their understanding with formal representations by translating the given text into mathematical notation (Sjuts, 2002). In all solutions, the students understand the meaning of the three successive natural numbers. The word "sum of" is interpreted as addition and the word "is" is an equal sign. They create a formula by using one variable. Each variable in each solution is explained as follows.

$n+(n+1)+(n+2)=93$ Solution 1	$y-4+y-3+y-2=93$ Solution 2
$n-2+n-1+n=93$ Solution 3	$3xm+3=93$ Solution 4

Figure 1. Students’ solution

In the first solution, the student uses the variable n to represent the smallest of three successive natural numbers. According to the student’s answer, the first number is one smaller than the second number, and the third number is one larger than the second number. The first number is represented by the term n, the second number is represented by the term $n+1$ and the third number by $n+2$. The placing of brackets clarifies the terms of the second and third natural numbers. Based on this setup, the student formalizes the problem and derives the equation $n+(n+1)+(n+2)=93$. In *the second solution*, another student employs the variable y, which is defined flexibly depending on the argument: y may be 4 smaller than the first number, 3 smaller than the second number, or 2 smaller than the first number. Consequently, the first number is expressed as $y-4$, the second number as $y-3$, and the third number as $y-2$. Using this representation, the student constructs a mathematical model resulting in the equation $(y-4)+(y-3)+(y-2)=93$. In *the third solution*, a student uses the variable n to represent the largest number among the three successive numbers. In this case, the first number is $n-2$ and the second number is $n-1$, leading to the formalization$(n-2)+(n-1)+n=93$. In *the fourth solution*, the student uses the variable m to represent the smallest natural number of the three successive numbers. Each summand contains the natural number m, i.e., three times the natural number m: $3m$. The second summand is larger by 1, and the third summand by 2. Therefore, the number 3 is added to$3m$. Thus, by combining the three summands and their differences, the student represents the sum of the three numbers as $3m + 3 = 93$.

Overall, the first, second, and third solutions demonstrate that the students write the sum of the three natural numbers in a way that all three numbers are explicitly noted, defining each successive number. The fourth solution shows that the student focuses on the first number alone and relies on the notion of succession to define the remaining two numbers. The four solutions presented by the students demonstrate that multiple forms of representation are possible in formalizing a problem, highlighting the practical value of developing formalization skills. Solution 1 is the most commonly observed approach, whereas Solutions 2, 3, and 4 are less frequent and require higher-order thinking. These less common solutions are more compressed and must be carefully analyzed to understand the meaning of the chosen variables and to correctly identify the successive natural numbers. The next step is students have to write down the solving process and the final solution.

Analysis of the Solving Process

In the solving process, the students manipulated the equations and provided justifications for each step.

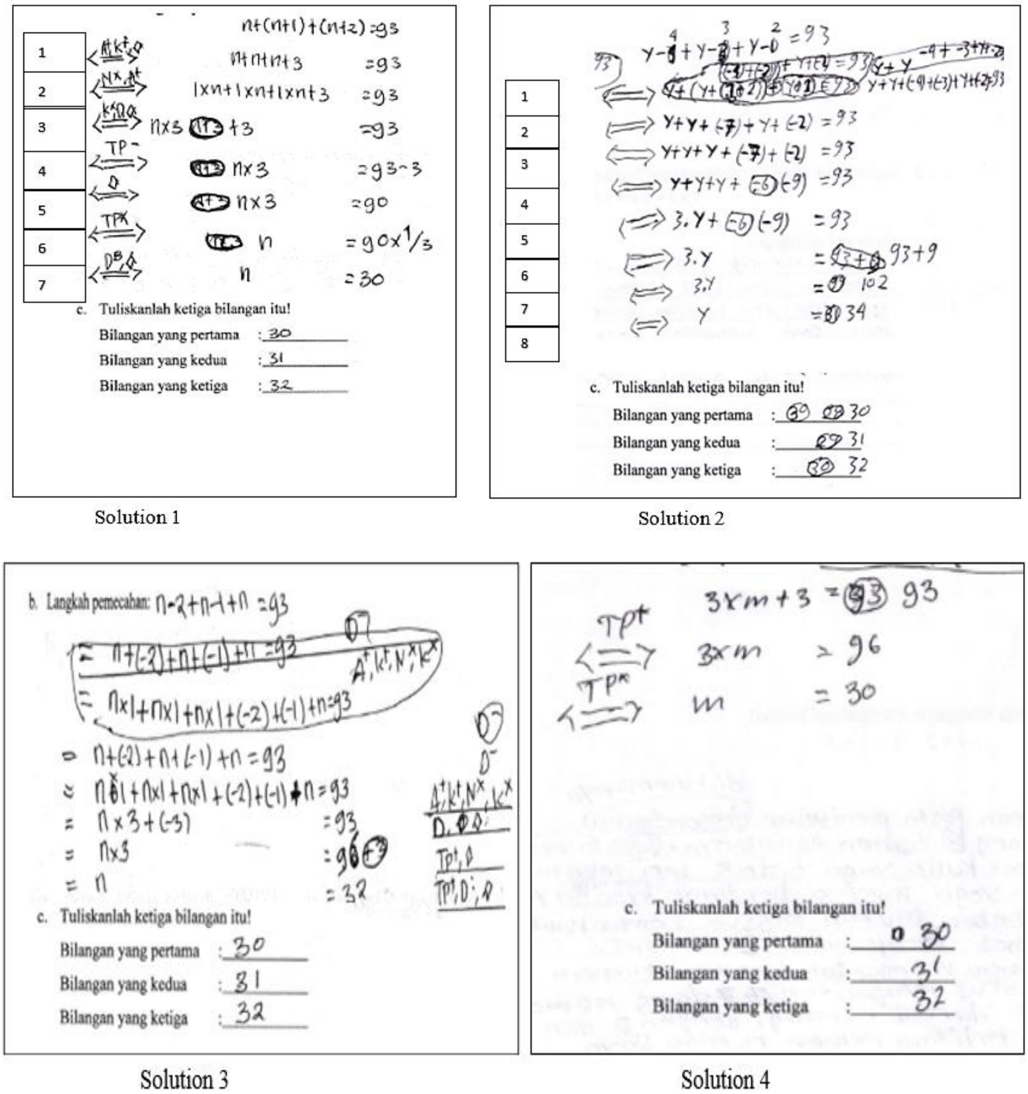

Figure 2. Students' solution

In *Solution Version 1*, the student explicitly writes the reason for every transformation. In lines 1, 2, and 3, the student applies the axioms of addition and multiplication. In lines 1, 3, 5, and 7, logical rules referred to as *Lupis* are used. The use of *Lupis* is widely accepted for justifying basic calculations learned in primary school, such as the sum of 1 and 2 yielding 3, or the division of 90 by 3 yielding 30. In lines 4 and 6, the student applies the theorem of equations for subtraction and multiplication, which they have previously studied and proven. At the final step, based on the formalization, the student determines the first number $n = 30$, the second number $n + 1 = 31$, and the third number $n + 2 = 32$. In *Solution Version 2,* the student does not write justifications directly within the main solution. However, on a separate sheet, the reasoning is provided. The process is mathematically correct, with each transformation accurately executed. Based on the formalization, the first number is calculated as $y - 4 = 34 - 4 = 30$, the second as $y - 3 = 34 - 3 = 31$, and the third as $y - 2 = 34 - 2 = 32$. The use of equivalence signs in Solutions 1 and 2 is consistent and correct. In *Solution Version 3*, the student writes the justification for each step. In lines 1, 3, and 5, definitions of subtraction, distribution, and division are applied. Line 2 applies the axioms of addition and multiplication, while logical rules are used in lines 3, 4, and 5. The student performs calculations such as $(-1) + (-2)$, $93 + 3$ and $96 \div 3$. In line 4, the theorem of equations on addition is applied to eliminate −3, and in line 5, the same theorem is used to eliminate 3; however, the stated reason is incorrect, as the student intended multiplication instead of addition. This is classified as a syntactic error. Additionally, the use of the equal sign on the left side throughout the solution leads to further syntactical errors. Despite these formal inaccuracies, the student demonstrates correct mathematical reasoning. The final solution, based on the formalization, correctly identifies the first number as $n - 2 = 32 - 2 = 30$, the second number as $n - 1 = 32 - 1 = 31$ and the third number as $n = 32$. In *Solution Version 4,* the solving process initially contains errors. In line 1, the student applies the theorem of equations on addition, whereas subtraction should have been used to eliminate 3 and produce 90 on the right-hand side. In line 2, the theorem of equations on multiplication is applied to eliminate 3, yielding 30. On a separate sheet, the student first identifies the variable $m = 30$ and then reconstructs the solving process. The final solution correctly identifies the first number as $m = 30$, the second as 31, and the third as 32.

Across Solutions 1, 2, and 3, the students rely on their prior knowledge, using established contractual reasoning to perform calculations, which they learned in Grade 7. Notably, in all four solutions, students marked incorrect steps by circling rather than erasing them. This practice, established in the classes, is intended to highlight errors and provide a basis for discussion, ultimately fostering deeper understanding.

Discussion

In designing the teaching and learning environment, giving priority to explaining and reasoning was considered essential. Although the test did not explicitly require students to provide justifications for their solving process, many students voluntarily offered explanations for each calculation step. By doing so, they applied axioms, definitions, logical rules, and theorems, demonstrating active engagement in reasoning rather than rote computation. According to Blum et al. (2002), such competencies are classified under Competency 5, which includes knowing and applying mathematical definitions, rules, algorithms, or formulas, and formally working with variables, terms, equations, or functions. Additionally, their ability to argue mathematically aligns with Competency 1. The students' initiative in providing justifications also reflects that their solutions go beyond simple instrumental understanding, as described by Skemp (1977, in Nowinska, 2014). In the absence of a fixed step-by-step plan, their reasoning indicates relational understanding and self-directed problem solving. These behaviors suggest that the teaching approach successfully encouraged students to articulate their thinking and reflect on their solution strategies. From a meta-discursive perspective, as outlined by Kaune (2018), the students' written justifications and explanations serve as evidence of developing metacognitive and discursive competencies. The classroom exercises were deliberately designed to promote reflection and reasoning, guiding students to internalize strategies and evaluate their own solutions. Exercise B, in particular, was intended to examine whether such training positively affected students' abilities to plan and execute strategies independently. Previous studies have shown that fostering learners' self-directed planning and strategic thinking is challenging (Depaepe et al., 2010). The examples of student solutions in this project demonstrate that, when supported with structured opportunities for reflection and justification, Grade 8 students can successfully internalize problem-solving strategies, articulate their reasoning, and apply formalization skills in a meaningful way.

Conclusion

The pupils' written solutions reveal that their mathematical understanding, through multiple interrelated competencies. They reasoned formally by applying axioms, definitions, and theorems, and they justified their procedures even when not required to do so. Their ability to plan and carry out strategies without relying on fixed procedures shows autonomous problem solving. The correct use of symbolic expressions and representations further demonstrates their conceptual clarity. Overall, these findings indicate that metacognitive-discursive teaching practices supported pupils in developing a deeper and more connected understanding of mathematics.

Recommendations

Since the analysis was limited to written test data, further studies should integrate classroom interaction analyses to capture how such competencies develop over time.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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