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How Do Junior High School Students Comprehend the Concept of Function?

Ika Wulandari
Indonesia University of Education

Sufyani Prabawanto
Indonesia University of Education

Abstract: Function is an essential material in mathematics as it is connected to real life very much and cannot be mastered easily, as shown with the findings of research on the learning difficulty faced by students in the material of function. This qualitative research with data triangulation is designed in case study aiming to explore learning obstacles based on students' comprehension on concept of function and how students construct meaning related to their learning experience. This research involves 13 (thirteen) 9th (ninth) graders of Junior High School in Gunungkidul Regency having received the material of function. Data were collected through test and interview. Result of interview was transcribed to be analysed in-depth to find out how the students comprehend concept and to identify potential learning obstacles. Result shows that overall, the concept of function is defined as: (1) correspondence between two elements; (2) there is inconsistent/partial definition of function; and (3) some students can define completely but limited to the context of linear function and square function. In conclusion, there is learning obstacle in the concept of function, including epistemological, ontogenic, and didactical. Some attempts or further research should be conducted to improve the learning of function to enable the students to comprehend completely through, among others: (1) completing the material with the concept map of function family tree; (2) conducting an analysis on the material of textbook using fraxeology; and (3) developing hypothetical learning trajectory and didactic design to overcome learning obstacle.

Keywords: Learning obstacles, Relations and functions, Function conception

Introduction

Mathematic learning paradigm in 21st century aims to give students strong reasoning ability. Therefore, NCTM (2000) recommends the learning oriented to how students master a concept, so that the concept can be internalized well by students (Fennema et al., 1996; Tan Sisman & Aksu, 2016) and strengthen the students' reasoning ability. The concept of function cannot be mastered easily, in addition to having essential material connected to real world (Bardini et al., 2014) and linked to many other mathematic concepts (Carlson M & Oehrtman, 2005; Denbel, 2015; Szanyi, 2015). The concepts connected to function are, among others number, equation, inequality, comparison, geometric transformation, trigonometry, calculus, and other concepts in mathematics. That makes the concept of function a bridge connecting one concept and another in mathematics. Even, the representation of function can be found in various pure and applied mathematics discussions (Ayalon et al., 2017). Therefore, if students have learning obstacle in mastering the concept of function, they will likely have obstacle as well in mastering other related mathematic concepts. Thus, the students' good concept comprehension in the material of relation and function is very important to be prioritized and the focus of current research. Students' success in comprehending and mastering the concept and its application in real problem can be an indicator of successful learning. A successful learning can also be determined by the learning process designed by the teacher. Designing learning is the most important task to teachers, as a foundation and guidance for a successful learning activity (Isman, 2011). Curriculum analysis conducted to determine lesson topic and theme to be delivered is the early step in making learning design (Suryadi, 2010). The next step is to make learning trajectory, to help prepare and

understand what is needed in the students' knowledge construction in the material of relation and function (Clements & Sarama J, 2008). In other words, teacher should master the material to be delivered. The more quality a learning process, the higher will be its ability of minimizing or overcoming learning obstacle potentially occurring.

Kansanen and Meri (1999) state that learning obstacle occurs due to the causing factors that can be seen based on teacher-student-material relationships. Learning obstacle will occur if at least one of the relationships is problematic. The intended learning obstacle, referring to Brousseau's (2002) and Suryadi's (2019) idea, is a condition making the students' process of acquiring knowledge inhibited during the learning process and thereby making students face problem in their learning. By its causing factor, learning obstacle can be divided into three categories: ontogenic (mental preparedness), didactical (teacher's teaching method), and epistemological (students' limited knowledge on contextual application)) (Brousseau, 2002; Suryadi, 2019).

Ontogenic obstacle is the learning obstacle occurring in relation to the students' thinking ability development. If the problem presented by teachers has difficulty level of too-easy or too-difficult, according to Suryadi (2019, p.7), ontogenic obstacle will likely be encountered by students. The obstacle arises when there is imbalance between learning target/objective and students' mental preparedness. It means that it is important for teacher to consider difficulty level of didactical situation in learning design containing task series in learning scenario/students' handbook. Didactical obstacle is the learning obstacle occurring related to the order of teaching materials. The obstacle arises when there is a mismatch between learning process occurring related to content and material presentation sequence, and the expected content and material presentation sequence theoretically (scholarly knowledge). This obstacle can be traced from teaching material/students' handbook used in the class.

Meanwhile, epistemological obstacle is the learning obstacle related to how students comprehend knowledge. This obstacle arises when students' knowledge on a material learned can be comprehended well in certain context but is unusable in other problem context. It means that teacher should pay attention to the context used in the learning to help students construct their knowledge to minimize the obstacle in the students. Although this material of relation and function is very important, the results of studies conducted in many countries showed that students still face many difficulties at each of educational levels in both Indonesia and other countries. Some studies found the difficulty in comprehending the concept of function in the students from secondary school to university. Sebsibe et al. (2019) found learning difficulty in the material of function in Senior High School students, involving limited mental representation about function approach, fragmented conception, and dependence on ordered pairs, limitation in algebraic manipulation, limitation in changing word expression into mathematic expression, confusing combination and composition, operation order unnecessarily exchanged when doing algebraic manipulation, and drawing graph regardless enough points. Another study conducted by Leinhardt et al. (1990) found that students have misconception on the lesson of function, related to 1) distinguishing function from non-function; 2) correspondence in the concept of function; 3) how to generalize the characteristics of linear function; 4) continuous and discrete graph; 5) various representations of function; 6) reading and relative interpretation; 7) concept of variable in function; and 8) notation in the graph of function.

Considering the problem above, a study should be conducted to investigate the learning obstacle the students encounter in the material of relation and function viewed from ontogenic, epistemological, and didactical obstacle categories, and to trace the the factor causing didactical obstacle originating from students' textbook. It is important because the students' textbook is one of supporting factors in comprehending material, in addition to learning process along with teacher in the class. The error and lack of material in textbook presentation potentially results in misconception among students. Therefore, this research aims to describe learning obstacles the students face based on students' comprehension in learning relation and function. With the information acquired, teachers will later make improvement in the next learning of function.

Method

The approach used in this research is qualitative one in the form of case study, through exploring the students' comprehension on the concept of function. The author served as primary instrument obtaining data from the subject of research, the 9th graders of Junior High School in Gunungkidul Regency, Yogyakarta, Indonesia. A test of function material was given to 13 (thirteen) students. The test consisted of 7 questions concerning the comprehension of function concept and its application to real life, with a design of time of 60 minutes for completing the test independently. Students' answers were checked and classified and then in-depth interview was carried out. The interview was conducted by giving open-ended question to 3 (three) students selected using purposive sampling technique based on the result of test review. The interview questions were composed in semi-

structured way to explore the students' test answer and experience with learning the concept of function.

This research focuses on exploring how the students' real conception on function, experience with constructing concept of function, and the potential learning obstacle occurring. There are two assumptions in this research: (1) each of students' responses contains a certain unit of meaning related to the concept of function; and (2) to find out the students' actual interpretation of function, the consistency of students' written and oral responses in the interview was observed. In addition, document study was conducted on the handbook used by students. Technique of analysing data in this research refers to the model suggested by Miles & Huberman (1992), in which following the data collection, the next step included data reduction, data display, and conclusion drawing or verification. The data reduction step was carried out by focusing data on important points in accordance with the objective of research. The data were then presented narratively to facilitate data comprehension, and then conclusion was drawn according to the problem statement studied.

Results and Discussion

Result and discussion of research findings are based on the specified assumptions that (1) each of students' responses contains a certain unit of meaning related to the concept of function; and (2) to find out the students' actual comprehension on the function, the consistency of students' written and oral responses in the interview is observed. The search for certain unit of meaning related to the function is adapted from the concept of function definition according to Vinner and Dreyfus (1989), including: (1) a correspondence of 2 elements; (2) an interdependence between two variables; (3) a rule of correspondence; (4) an operation or manipulation; (5) a symbolic representation: formula, algebraic expression, equation; and (6) a representation: verbal, graphic, numeric. Overall, the discussion of research finding involves: (1) students' conception on function, (2) students' experience with constructing the concept of function, and (3) learning obstacle the students likely encounter.

The first test question displays the graph of body weight in the grow chart (Indonesian: Kartu Menuju Sehat or KMS) with a child's original data. The students were told to examine and to explain the curve of body weight depicted in the KMS, whether it is a function or non-function. Out of 13 students, 7 students state that it is a non-function for the reason related to the result of domain reading considered multifaceted or having more than a partner in codomain. Meanwhile, the domain data constitutes age from zero to 14 months, and the codomain data constitutes body weight in kg. Thus, it does not matter if the body weight (range) is the same in different months. The display of KMS data putatively generates confusion or difficulty for the readers not accustomed with filling in the display data in KMS. Moreover, it will be more confusing when the data put on X (domain) and Y (codomain) axes are integer but there is a note added manually constituting the actual weight in the form of decimal fraction below the data of month (manual recording of name, months when the weight begins up to the next 14 months) on X axis. Thus, the careless students will misread and consider that the data of actual body weight is the domain data. The display of ideal weight parameter curve, addition to actual weight, will also result in distinctive confusion or difficulty to the students in reading the data. Meanwhile, the other 6 (six) students could detect that the graph of body weight development in the KMS is a function, seeing that the members of domain paired exactly with the members of codomain. The result of test and interview to search for the definition of function concept for the question number 1 is presented in the Figure 1.

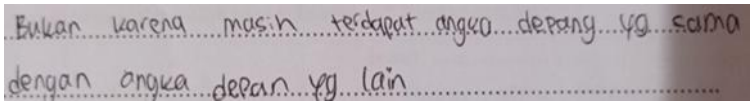


Figure 1. The first student (S1)'s answer to the question number 1

Considering the result of interview with the first student (S1), it can be seen that the student identifies the meaning of function as the rule of correspondence between two elements, meaning (1). The student identifies the interrelation between domain (front) and range located in codomain (back). In addition, the student also identifies the rule of relation between domain and codomain, that domain may not have more than one pair in the members of codomain. However, the students misconceive that domain is always on X axis or the sets located at the front while codomain is always presented on Y axis or sets located at the back. Students do not pay attention to mapping direction. It is possible because the habit of memorizing without comprehending the condition given in the question. The exploration of textbook used by students actually has elaborated and given the example of mapping from set A to set B, and vice versa, but it has not been balanced. The discussion on the mapping from set A to set B is more dominant, so that the author saw this potentially results in didactical obstacle. In addition, in a interview related to the answer to the question number 1, it could be seen that the first student (S1) misread the data of weight containing decimal fraction number, and assume it as points (x, y). It is putatively because the student was

accustomed with reading numeric data containing decimal fraction. Generally, in the material of Cartesian axis, question or discussion in the textbook address's integer only. It makes the students unskilful in the exploring or constructing the graph of function with input of decimal fraction number. Furthermore, the question number 2 is presented in a variety of curve images, and then students were told to determine and to explain whether or not the curve is the function from x to y, as shown in Figure 2 below.

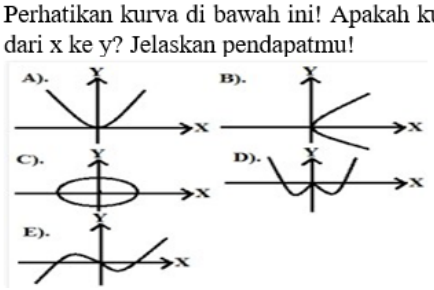


Figure 2. Question number 2

Nearly all out of 13 students cannot identify curves B and C as non-function. The students also find difficulty in determining curve D as a function or a non-function. For the Curve B, some students state that the curve is a graph of function and inform that the graph has been exemplified in the textbook.

From the question number 1, the student identifies the meaning of function as the rule of correspondence between 2 elements (3), the student also knows the relation between domain and codomain in the graph of function. In solving the question number 2, the student still identifies graph A consistently as a function recalling that it has been exemplified in the textbook; then, through scaffolding in the interview, the student can see the relationship of domain to codomain in which domain should pair exactly with the member of codomain. However, the student has not been able to explore consistently the relation of domain to codomain in other graphs. This is putatively,

Q : Ok, try to explain what information you got from question number 2?
S1 : I was told to see which graph is a function or not, Madam....
Q : How do you think it should be done?
S : I forgot it, Madam, but it seems like Figure A shown in the book. So maybe the graph of A is a function, but I have not seen the others in the book.
Q : Oh... it is okay. Earlier when you were working on number 1, you said that the x-axis always serves as domain, right? And the y-axis always serves as codomain. Now, pay attention to the x-axis as domain, and the y-axis as codomain. What is the relationship between x and y on the graph?
S1 : um ...(thinking for a moment)... All the x-axes in Figure A are different Madam, none of them have double pairs
Q : Oh, okay. Now, let's look at another figure...
S1 : Figure B is also shown in the book, Madam, but I forgot it... I do not know the others, Madam . I am confused looking at it... it makes me dizzy, Madam.. hehehe

due to the students' limited experience with exploring various forms of graph, either as the example or the exercises to distinguishing the graph of function from that of non-function. The result of interview with S1 is cited below to explore the answer to the question number 2. Then, the question number 3 presented various diagrams, and the students were told to determine and explain whether or not the curve is the function from x to y, as shown in the Figure 3.

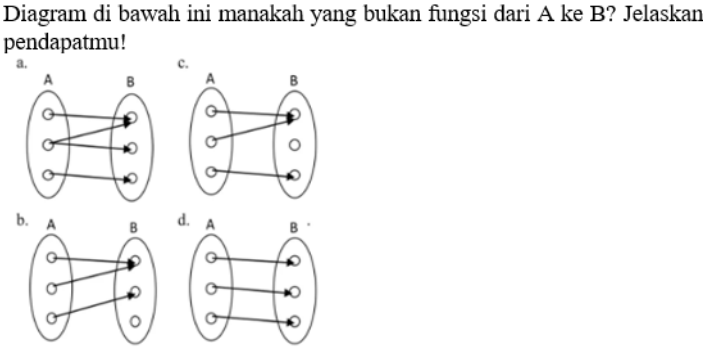


Figure 3. Question number 3

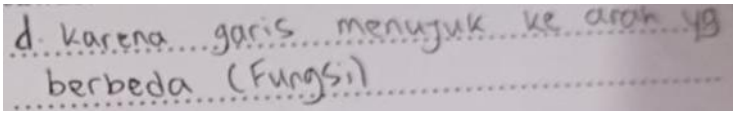


Figure 4. The first student (S1)'s answer to question number 3

The terms memorized by the student as "the front may not be double", in fact generated confusion (ambiguity) inside the students so that they found difficulty in distinguishing diagram of function from that of non-function. Then, the students could distinguish the diagram of function from that of non-function, when their language use was corrected with the following sentence: "domain may not have more than one partner" or "domain has one partner exactly in codomain". Such condition is shown in the result of interview cited below.

Q : For question no. 3, your answer is d, which is not a function. Can you explain why?
S1 : I meant that d is the function, Madam... I gave an explanation in brackets at the back.
Q : Oh I see… Okay. What does it mean when the lines point in different directions?
S1 : So, part A is paired with B, all of them are different, none of them are the same. So, this is the function, Madam.
Q : What about diagrams a, b, and c? Are they functions or not functions?
S1 : It is not a function, Madam... because there are some that have more than one pair of arrows ... there are double pairs
Q : Okay... so which one cannot have more than one pair, a domain or codomain or both?
S1 : .....um… it is the front one, Madam… the domain
QP : Okay..., try to look again at the diagrams of parts a, b, and c... what is the relationship between domain and codomain?
S1 : …. (pauses for a moment)… diagram a has members that have 2 pairs in the codomain, so a is not a function Madam…. Meanwhile, for b and c… the members of the domain have one pair, Madam, none of them have more than 1 pair… so b and c are functions, Madam...

In contrast, the result of analysis on answer and interview with the second student (S2) for question numbers 1 to 3 shows that the student identified the meaning of function as the rule of two-element correspondence (1) and could display it in various representations, meaning (6). The student identified the relation between two sets and then could identify the graph in KMS as the function to collect the data of domain member and its partner in the table. The student ensured the use of numeric representation in the form of table data to see whether or not the domain has one partner exactly in codomain. The student also recognized that the representation of function in the form of graph can be displayed in other forms: table, ordered pair set and function diagram. The explanation can be seen in the answer to question in Figure 5 and the result of interview cited below.

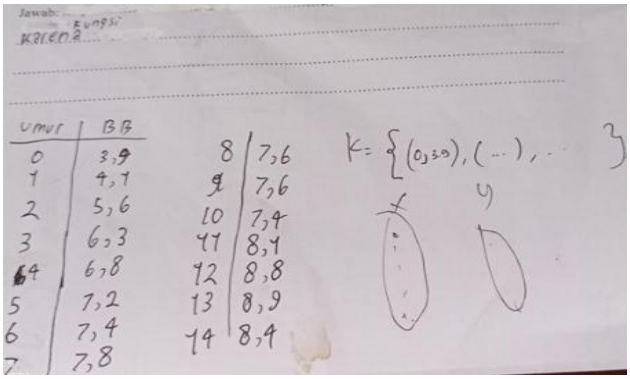


Figure 5. The second student (S2)'s answer to Question number 1

Q : In your answer number 1, the KMS graph is a function graph, why?
S2 : It is because if you make a table, you can see that the members of the age set have exactly one partner in the weight set.
Q : Ok... try to pay attention, there are 2 members with the same data of weight: 7 and 6 , and 8 and 4... How do you explain it?
S2 : It is okay, the one that has to be paired is the one in front.
Q : Ok, can this data be presented in another form?
S2 : Yes, Madam.. it is like this… (writing a set of ordered pairs, and making a diagram of the sets X (age) and Y (Weight))

Furthermore, considering the result of interview with the second student (S2) and the answer to question numbers 2 and 3, the second student (S2) still consistently defined function as the correspondence relation between 2 elements. But the second student (S2) found difficulty in recognizing certain graph as a function or non-function.

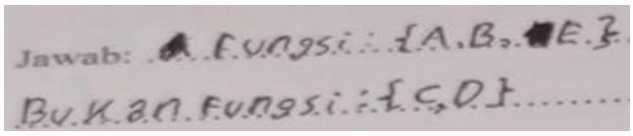


Figure 6. The second student (S2)'s answer to Question number 2

The student identified graphs A and B as a function recalling that it has been exemplified in the textbook, and then graph E is categorized into function as it is considered as the combination of 2 function graphs. The student did not identify graphs C and D as he thought that he had never seen it in the textbook.

Q : Try to explain why graphs A, B, and E are functions and graphs C and D are functions.
S2 : There are examples of A and B in the book, Madam... they are functions... E is a combination of 2 function graphs... I have never seen C and D, Madam.
Q : Try to pay attention again, ... how is the relationship between domain and codomain?
S2 : ... if we make a table… the x part cannot have double pairs…but there are no numbers written here, Madam…so it is not obvious
Q : Can we ascertain it without knowing the numbers?
S2 : …Yes, we can Madam… maybe we can (looks doubtful)
Q : Ok, now try to pay attention to them one by one.
S2 : … A is clearly a function… every x has 1 partner in y, Madam…
Q : Okay, now look at B...
S2 : …. If I take 1 x… there are 2 pairs in y: positive y and negative y… meaning that B is not a function, Madam… C is also not a function
Q : Alright…… what about D and E?
S2 : D and E are functions... So is my answer wrong, Madam?
P : What did you think of first when you read this question?
S2 : I remember what is mentioned in the printed book, Madam... I did not try to work on it because there were no numbers on the graph.

The student found difficulty in exploring the relation of domain to codomain because the graphic presented in the question did not contain the information of points in the graph and there is no numeric signs on X and Y axes. Other students also encountered this, one of which is the third student (S3) in the question number 2. The following is the answer and citation of interview with S3 for the question number 1 to 3.

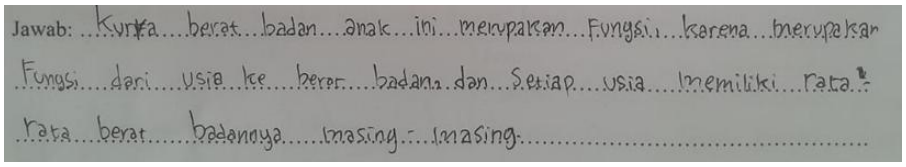


Figure 7. The third student (S3)'s answer to Question number 1

The third student (S3) defined function as the correspondence relation between two elements. However, from the result of interview, it can be seen that in facing this question, the student tried to imagine the form of curve, including linear or square forms, and then thought of its general formula. In the interview for the question number 1, it can be seen that the student defined function as an algebraic expression or an equation, meaning (2). Just like S2, the third student (S3) also paid inadequate attention to the direction of mapping. It is putatively because the students' handbook did not facilitate this experience proportionally. Similarly, in exploring various forms of curve and determining whether it is the function from x to y, S3 showed his consistency by defining function as an equation or algebraic expression, meaning (5).

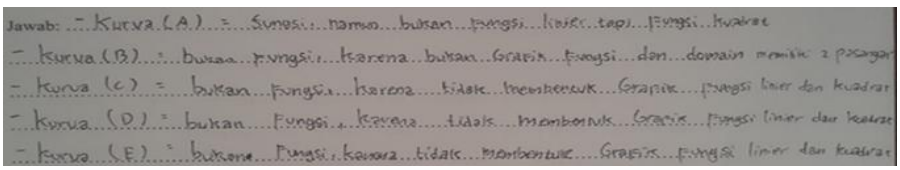


Figure 8. The third student (S3)'s answer to Question number 2

Q : Please explain your answer to number 1!
S3 : Okay, I am ready,Madam... the KMS graph is a function of age and weight. And each domain has its own weight.
Q : Which one?
S3 : the one on the X axis... age is domain... y axis is codomain... the range is body weight for each age
Q : Some of them have double weight... 7 and 6 appear twice... so it is double...
S3 : That is the range of codomain, Madam... so I think it it is okay...
Q : Oh yeah… Is the domain always on the x-axis?
S3 : Yes it is, Madam... it is usually on the x-axis
Q : Is it possible to reverse the function of weight versus age ?
S3 : Yes, it is, Madam... but I am not sure... because I do not understand the general formula for this graph...
Q : What was the first thing that came to your mind when you read this question?
S3 : I was thinking about what the general formula is, Madam... it is just like a quadratic function graph that has an irregular shape.

Each time faced with the problem of function, S3 was always oriented to finding the equation first. S3 at least wanted to find out the general form of problem.

Q : Okay… how about question number 2… what was the first thing that came to your mind when you read this question?
S3 : I kept remembering what the general formula is, Madam...
Q : What kind of the general formula?
S3 : The one stating that A is the graph of the quadratic function y = ax2+bx+c… B is not a function because its domain x has more than one pair…
Q : Okay, then what about C…
S3 : Looking at this C, I thought, why is the graph from C to D and E... starting to look strange... it is neither a linear function graph nor a quadratic function... so I think it is not a function.
Q : Oh, so if there is a graph that is not a quadratic equation or a linear equation, it will mean that it is not a function, right?
S3 : Yes, Madam... it is because as far as I know, the two are the one having been studied, the linear one is f(x)= ax+b... or y=ax+b...
Q : Take another look at C... see whether or not the domain has exactly 1 pair in the codomain? Try to take one value of x.
S3 : Oh okay, Madam... there are 2 ys: positive and negative... so C is not a function
Q : All right… now pay attention to D and E…
S3 : E seems to be a function… but I am not sure… like I have seen it before… maybe a cubic function or something, I have forgotten its name
Q : What about D?
S3 : Um… I cannot decide what function it is… I have never seen it before… I am not sure, Madam…
Q : Have such questions never been discussed?
S3 : They have never been mentioned in the book, Madam... if there are challenge questions in the book, they are usually skipped and not explained...
Q : All right... from these pictures... we are told to find which is a function of x to y... can it be reversed ... function of y to x?
S3 : um… yes it perhaps can, Madam… it depends on the person who made the questions
Q : Try to pay attention to A and B... are they a function of y to x?
S3 : Um… the general form is y=ax2+bx+c… it maybe can be reversed to x=… but I am not sure
Q : Can't it be seen from the relationship between domain and codomain alone?
S3 : I still adhere to the general formula form as my basis, Madam.

The third student (S3) defined function not only as comprehending the relation of 2 elements but also comprehending various representations of function, including generalization in the form of symbolic expression or determining the equation formula. However, for this question number 2, the student admitted that he has not been accustomed with solving graphic question without information on domain number input (x axis) and its range (y axis).

Meanwhile, nearly all students answer correctly the question numbers 3 and 4. S3 stated that the question number 4 is the easiest one, because domain and range numbers are written clearly. This question presents 4 ordered pair sets, then the students were told to determine which one is the function from x to y. Out of 13 students, only 2 students did not answer, the rest answered correctly.

Furthermore, the questions number 5 and 6 present algebraic expression equation implicitly. Out of 13 students only 2 students could answer them. Meanwhile, the rest could not answer them or stated that the equation is non-function. The students argued that they have never done or found such question either in the class or in the textbook. The question numbers 5 and 6 are presented in Figure 9.

5	Apakah $x + y = 1$ merupakan fungsi dari x ke y? Jelaskan pendapatmu!
6	Apakah $x^2+y^2=1$ merupakan fungsi dari x ke y ? Jelaskan pendapatmu!

Figure 9. Question number 5 and 6

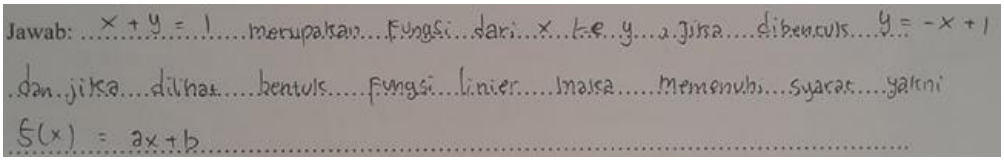


Figure 10. The third student (S3)'s answer to Question number 5

Q : For question number 5, what did came to your mind first?
S3 : I focused on the equation, Madam... if it can be changed into the general form of a linear equation y=ax+b or f(x)= ax+b.... that means the function... becomes y = -x+1
Q : To find out the relationship between domain and codomain… what can be done to explore this issue in depth?
S3 : After I was sure of the formula... I would immediately draw the graph, Madam...
Q : Oh I see … isn't it easier to record the data in a table and then draw the graph?
S3 : Even though I was always taught that way… I did not go through that method, Madam… because it took too much time to create the data in that table…

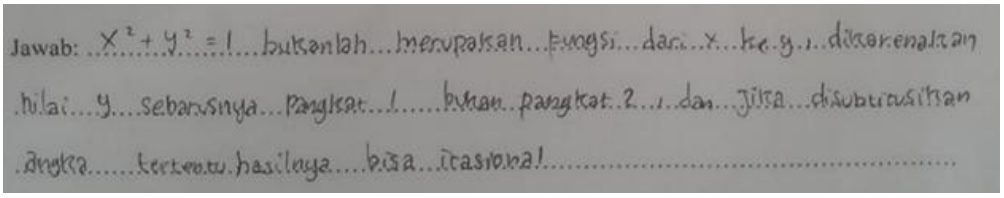


Figure 11. The third student (S3)'s answer to Question number 6

Q : What about question number 6? What did came to your mind first?
S3 : I was looking at the formula, Madam... this is not a function... the formula cannot be transformed into a linear or quadratic equation. It will become a negative square root if it is forced to be transformed into the form y = ...
Q : How about listing the domain (x) and its partner (y)
S3 : Um…. Suppose x=0… y=1… is that right, Madam?
Q : If x=0… does y only have a value of 1?
S3 : Yes, it does, Madam
Q : (-1) x (-1)= … how much?
S3 : 1, Madam
Q : So? If x=0… what will be the possible values of y?
S3 : Oh yeah… it could be 1 or -1…
P : So there are domains with more than one pair, right?
S3 : Yes, Madam... so it is not a function... I cannot think of any other reason, Madam... I focused on the formula... if I do not recognize it as a linear equation or a quadratic equation... I will not think that it is a function...

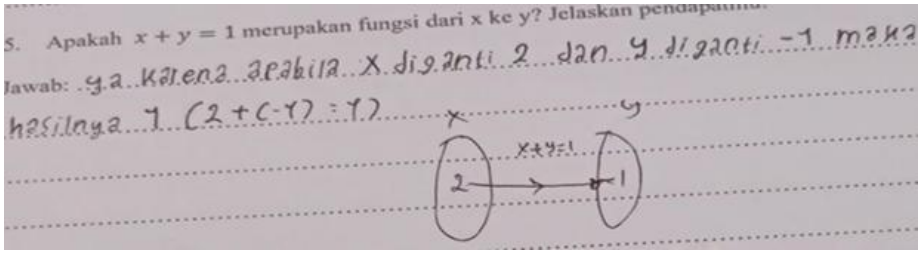


Figure 12. The second student (S2)'s answer to Question number 5

Q : What about number 5? Is x+y=1 a function?
S2 : Actually, I am doubtful... but if I take the number 2 as an example to be inserted into x, it will mean there is y=-1 as a pair of x... etc ... so we can make a table and draw a diagram of the function. So I think this is a function
Q : Have you ever encountered a question/exercise with the form x+y=1 like this before?
S2 : I have never encountered it Madam… it is usually f(x)=….
Q : When you saw this question, what did you think of first?
S2 : I thought if this is a function like the usual f(x)= or y=… it will mean that there is an x value that can be entered and produce y as usual… Well, if you do not find the result, it will mean it is not a function.

The second student (S2) has not experienced yet with exploring the symbolic representation in the form of algebraic expression implicitly. However, the students could use his learning experience through trying to find input and output values in the implicit formula of function. In this question number 5, S2 defined function as meaning (4), operation or manipulation.

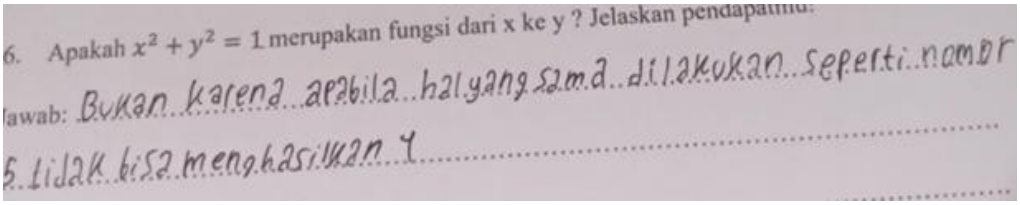


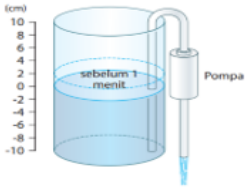
Figure 13. The second student (S2)'s answer to Question number 6

Q : What do you think about number 6?
S2 : I have never known this either, Madam. But if I take a number for x, it will not produce y.
Q : What is the number, for example?
S2 : If x is 1, it will be $1^2 + y^2 = 1$; that is not possible, Madam...
Q : Think about it again, doodling is allowed... 1+how much = 1?
S2 : it is zero, Madam, that means there is a partner, Madam... it means that it is a function
Q : If this is a function, what should the domain pair be like?
S2 : The front part only has 1 pair, Madam... so the x cannot have a double pair.
Q : Do you think numbers 5 and 6 are definitely functional, just by looking at 1 pair of data?
S2 : If you have a partner, it is clearly a relationship... but I do not know yet, maybe there is a double x, Madam.
Q : So what should we do next?
S2 : We should record as much data as possible ,… until we know whether or not there are any duplicates
Q : try to think of other data that might be obtained…
S2 : If x is 2, it will be $2^2 + y^2 = 1$; this cannot be a negative root, how can it be Madam... x cannot be a number 2 or above, it is not possible
Q : Is it allowed to input x or y as a fraction? Rather than an integer, for example, a fraction below 1...
S2 : um… I did not think about that, Madam… as far as I remember, when I was studying from the book, the numbers were always right… but it seems like it should be allowed.
Q : Okay... earlier you wanted to look for data that shows that there is an x that has a double pair in y, right ?... try to check if x=0, what will be the value of y?
S2 : if x is 1, it will be 02+ y2 = 1 so $y^2 = 1$, y=1
Q : $(1)^2 = 1$ right ?... what about $y^2 = (-1)^2 = ...$
S2 : it is 1 too… it means that when x = 0 the value of y can be 1 or -1, so it is double… this is not a function…

The student consistently used his/her learning experience by trying to find input and output values in the equation presented implicitly. In the question number6, S2 consistently defined function as the meaning (4), operation or manipulation. But, S2 had difficulty in finding some value pairs (x,y). It is because S2 was less experienced with exploring mathematic equation with fraction input and negative number.

Q : What do you think about number 6?
S2 : I have never known this either, Madam. But if I take a number for x, it will not produce y.
Q : What is the number, for example?
S2 : If x is 1, it will be $1^2 + y^2 = 1$; that is not possible, Madam...
Q : Think about it again, doodling is allowed... 1+how much = 1?
S2 : it is zero, Madam, that means there is a partner, Madam... it means that it is a function
Q : If this is a function, what should the domain pair be like?
S2 : The front part only has 1 pair, Madam... so the x cannot have a double pair.
Q : Do you think numbers 5 and 6 are definitely functional, just by looking at 1 pair of data?
S2 : If you have a partner, it is clearly a relationship... but I do not know yet, maybe there is a double x, Madam.
Q : So what should we do next?
S2 : We should record as much data as possible ,... until we know whether or not there are any duplicates
Q : try to think of other data that might be obtained…
S2 : If x is 2, it will be $2^2 + y^2 = 1$; this cannot be a negative root, how can it be Madam... x cannot be a number 2 or above, it is not possible
Q : Is it allowed to input x or y as a fraction? Rather than an integer, for example, a fraction below 1...
S2 : um… I did not think about that, Madam… as far as I remember, when I was studying from the book, the numbers were always right… but it seems like it should be allowed.
Q : Okay... earlier you wanted to look for data that shows that there is an x that has a double pair in y, right ?... try to check if x=0, what will be the value of y?
S2 : if x is 1, it will be 02+ y2 = 1 so $y^2 = 1$, y=1
Q : $(1)^2 = 1$ right ?... what about $y^2 = (-1)^2 = ...$
S2 : it is 1 too… it means that when x = 0 the value of y can be 1 or -1, so it is double… this is not a function

Pada gambar di samping, dari tangki yang terisi penuh setinggi 20 cm, air dikeluarkan dengan pompa. Ketinggian air berkurang 2 cm per menit. Misalkan, 0 adalah titik acuan, dan y cm adalah ketinggian air setelah x menit.



Perhatikan gambar di bawah ini. Berdasarkan kondisi di atas, kerjakan soal-soal di bawah ini!

a. Isilah titik-titik di bawah ini, perhatikan hubungan antara x dan y.

x (menit)	-5	-4	-3	-2	-1	0	1	2	3	4	5
y (cm)					2	0					

b. Dapatkan kita simpulkan bahwa y berbanding lurus dengan x? Jelaskan alasanmu.
c. Apakah nilai y naik ketika x naik? Ataukah turun?
d. Jika air dalam tangki yang semula penuh, dipompa dikeluarkan 3 cm per menit, Nyatakanlah y dalam x menggunakan persamaan?

Figure 14. Question number 7 about function application

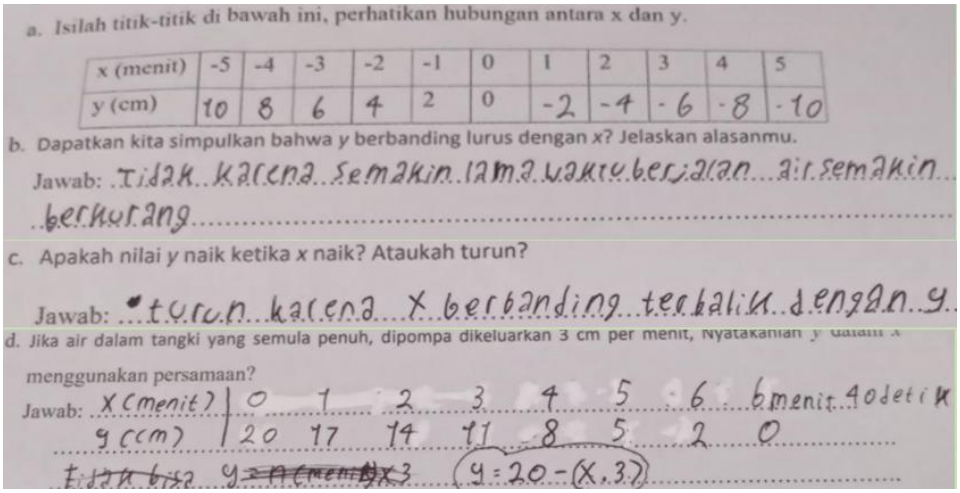


Figure 15. The student (S2)'s answer to Question number 7

Q : What did you think first when you read this question?
S2 : This must be a function because the contents of the x table are all different, if you fill in y it will also all be different.
Q : Does this question feel easy or difficult?
S2 : When I read the problem, it feels like simple calculations so it can be done manually... but seeing the picture with measurements from 10 to -10 makes it difficult, Madam... imagining negative time makes your mind confused.
Q : How complicated is it?
S2 : connecting negative time and height, there are also negative ones from the zero reference in the middle, but filling in the table is easy.
Q : Okay… what is the difference between directly proportional and inversely proportional?
S2 : Directly proportional means if x increases, y also increases... if it decreases, it will decrease.
Q : Oh I see… then how do you solve number 7 point d?
S2 : It will be easy if we make a table, Madam, so we can see the relationship between x and y
Q : Why is your table x starting from zero? And the height is 20 cm, without using the picture illustration in the question?
S2 : If we follow the picture, it is hard to imagine, Madam... so I will just make it easy... all the numbers are positive... so it is easy to make the equation.

In this question number 7, S2 consistently defined function as the meaning (4), operation and manipulation. S2 consistently used his/her learning experience by trying to collect data of input and output values from the contextual problem existing to facilitate the construction of function equation. Seeing the numeric representation in the data of table form, S2 could identify and believe something as function or non-function. However, S2 found difficulty in remembering the concept of direct comparison and inverse comparison, constituting one of prerequisite materials to learn function.

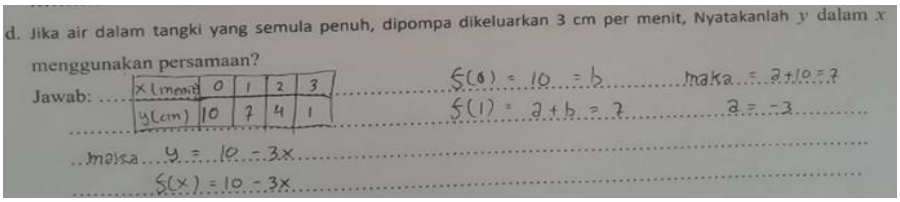


Figure 16. The third student (S3)'s answer to question number 7

The third student (S3) consistently defined function as the meaning (5), an equation or algebraic formula. The student consistently used his/her learning experience by trying to collect data of input and output values from the contextual problem existing to facilitate the construction of function equation. Having constructed the equation completely, the third student (S3) could only identify this problem as the applied problem of linear function. Also, S3 could express his/her comprehension on the concept of function.

In several cases, the difficulty of comprehending the concept of function encountered by the students was caused by limited context and learning experience with exploring diverse problems of relation and function. The source of context limitation is, among others, textbook as the students' learning source, in which error and less diversity are found in the presentation of relation and function material, including: (1) the error of non-function example presentation in students' textbook'; (2) the error of function graph presentation related to continuous or discrete function; (3) the example of discussion of questions and exercise questions in textbooks emphasizing various essential concepts for students' learning experiences including, among others: (a) contextual question containing the real-world problem solving; (b) diversity of relation and mapping direction; (c) presentation of equation in implicit form; (d) presentation of various linear, square, and non-linear curves to explore the representation of function and non-function graphs.

The error of non-function example in students' textbook, Table.35, example number 8, page 98.

The process of constructing knowledge inside students is highly dependent on their learning experience. The students' learning experience is closely related to textbook and teacher. Inadequacy and error in material presentation in the book will make students doubt the previous knowledge and lack of diverse and intact learning experience. Students' doubt/confusion with essential concepts in the beginning of material will generate obstacle in further comprehension, if it is not corrected immediately.

Q : Is question no. 7 easy or difficult?
S3 : Insha Allah, it is easy, Madam... there is just something that is a problem..., that is, the negative minutes and why the zero reference is in the middle... why it is not just at the top...
Q : Does it feel complicated because of that problem?
S3 : Not really, Madam... but I tend to want to avoid it.
Q : How do you answer questions a to d?
S3 : I filled in the tables one by one… yeah, it is just as usual, Madam…
Q : What is the difference between directly proportional and inversely proportional?
S3 : directly proportional means if x increases, y will also increase, while inversely proportional means if x increases, y will actually decrease...
Q : Are equations showing proportional (directly proportional) and inversely proportional ratios also functions?
S3 : I never thought about it, Madam... I cannot imagine it
Q : Okay… this is data a, earlier it was said to be an inverse comparison, right? Look at the domain and its pair… is it a function?
S3 : Oh.. yes it is the function, Madam... I did not think about it, Madam... because I always focus on what the similarities are ... if the equation has not existed... I did not think about anything else.
Q : Oh... Okay... for the d... why does the data defined for its minutes start from zero, then the reference is 10... why it is not 20 when the water is full, or zero in the middle if you look at the conditions in the illustration
S3 : I avoid the problem of negative minutes and find an easy way to make the equation, Madam.
Q : All right, now look at the equation we got… f(x) = 10-3x. So, from this equation, do you think the graph is a straight line or a curved curve? Function or not function?
S3 : It seems like a linear equation, Madam... f(x) = ax+b... that means that it is a function, Madam.
Q : If you look at the equation and graph, does that mean it is directly proportional (equal) or inversely proportional?
S3 : It seems to be directly proportional, Madam…
Q : What is memorable about learning relations and functions from start to finish?
S3 : One-to-one correspondence is definitely a function, a function is definitely a relation, but a relation is not necessarily a function, and a function is not necessarily a one-to-one correspondence. A function can be expressed as f(x)= , a linear or quadratic equation, which can be inputted with the value of x, resulting in y. It can be made into a graph, a set diagram, or a table.

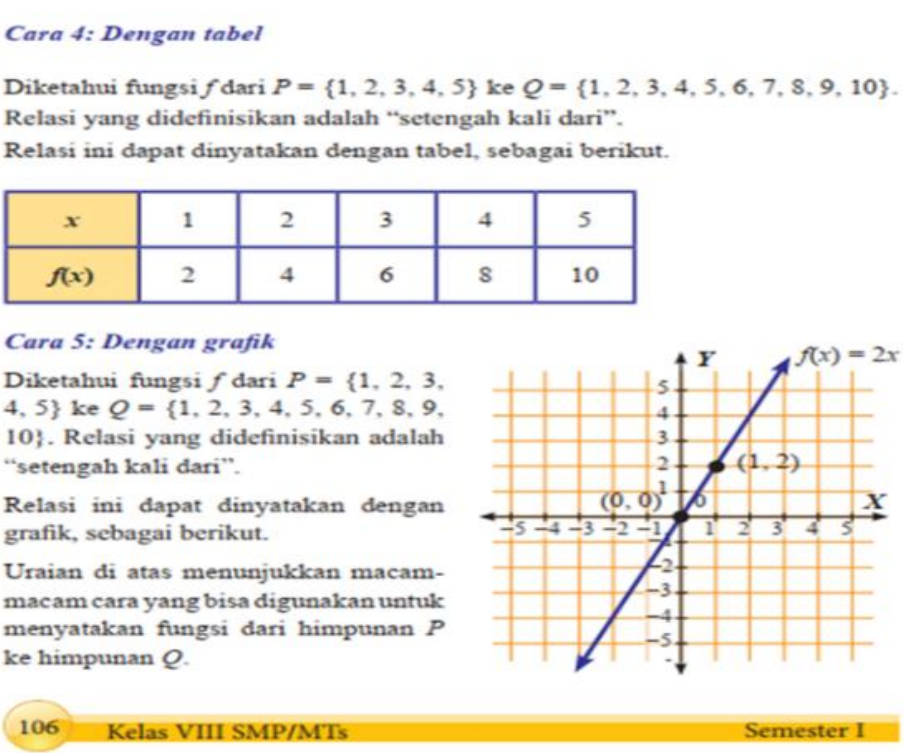
Tabel 3.5 Contoh fungsi dan bukan fungsi

Contoh Fungsi	Contoh Bukan Fungsi
1. $\{(1, a), (2, a), (3, a)\}$	1. $\{(1, a), (2, a), (2, b)\}$
2. $\{(1, b), (2, b), (3, b)\}$	2. $\{(1, b), (2, a), (2, b)\}$
3. $\{(1, a), (2, a), (3, b)\}$	3. $\{(1, a), (1, b), (3, b)\}$
4. $\{(1, a), (2, b), (3, a)\}$	4. $\{(2, a), (2, b), (3, a)\}$
5. $\{(1, a), (2, b), (3, b)\}$	5. $\{(2, a), (2, b), (2, c)\}$
6. $\{(1, b), (2, a), (3, a)\}$	6. $\{(1, b), (2, a), (2, b)\}$
7. $\{(1, b), (2, b), (3, a)\}$	7. $\{(3, a), (3, b), (3, c)\}$
8. $\{(1, b), (2, a), (3, b)\}$	8. $\{(1, b), (2, a), (3, b)\}$

Figure 17. Error of example presentation in textbook

In addition to the error of example presentation, the error of function graph presentation is also found. The data of domain, codomain, and range should be written carefully, and students are emphasized to criticize them precisely. Based on the author's experience as a teacher, if it is not corrected, it will generate obstacle continuously in the more complex material of function. It should be emphasized that the data should be written correctly, whether it is discrete or continuous. If the data is discrete, its graphic/visual representation can be points only without connector line. On the contrary, if the data is continuous, the graphic/visual representation can be line connecting an infinite number of points.

The observation of domain data, either discrete or continuous, should be emphasized on either formal mathematic problem solving or the problem related to the application of function in real world. The students' doubt or even ignorance in this case can result in an error of problem solving and misconception. Not all problems in daily life are continuous or discrete function. As a result, not all problems can be represented using continuous or discrete function graphs, meaning that the solution to problem is different. In addition, the careless and ignorant students will find difficulty in seeing domain input in the continuous function graph, if the domain input is fraction number. The students are usually unaware that there is infinite point on the line in continuous graph as a solution to a problem of function.



Cara 4: Dengan tabel

Diketahui fungsi f dari $P = \{1, 2, 3, 4, 5\}$ ke $Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
Relasi yang didefinisikan adalah "setengah kali dari".
Relasi ini dapat dinyatakan dengan tabel, sebagai berikut.

x	1	2	3	4	5
$f(x)$	2	4	6	8	10

Cara 5: Dengan grafik

Diketahui fungsi f dari $P = \{1, 2, 3, 4, 5\}$ ke $Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Relasi yang didefinisikan adalah "setengah kali dari".

Relasi ini dapat dinyatakan dengan grafik, sebagai berikut.

Uraian di atas menunjukkan macam-macam cara yang bisa digunakan untuk menyatakan fungsi dari himpunan P ke himpunan Q.

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Figure 18. Error of function graph presentation

Then, in addition to the error of example and function graph presentation, there is also a weakness or limitation in the presentation of question example and exercise in textbook. The presentation gives less emphasis on diverse essential concept for the students' learning experience and thereby generates learning obstacle in the students, as shown in the Table 1 below.

Conclusion

Considering the result of research, overall the concept of function is defined a (1) correspondence of two elements; (2) there is inconsistent/partial definition of function; and (3) some student could define it completely but limited to the context of linear function and square function. In addition, learning obstacle occurring in the students can be found, including epistemological, ontogenic, and didactical. Ontogenic obstacles include: (1) conceptual obstacle: partial definition, students not ready for learning the function further because of inadequate mastery of basic concept of function (prerequisite) in the materials of sets, coordinate position, and comparison; (2) psychological obstacle: obstacle in the form of disinterest in learning mathematics, particularly in the material of function. Out of 13 students taking the test, 6 stated that they are less interested in mathematics, particularly the material of function.

Table 1. Data of exploration of inadequate presentation in student textbook

No	Type of Exercise	Weakness of Textbook Presentation	Impact/ Obstacle
1.	▪ Exercise 3.1 ▪ There are 15 questions presenting relation	Only 3 simple contextual questions do not need real problem solving	▪ The advantage of relation and material function in daily life is perceived poorly by the students. If the students learn something difficult and felt less and even not advantageous, it will have an impact on the students' decreasing interest. ▪ Students' experience not emphasizing the exploration to see the direction of relation/mapping makes their comprehension incomplete.
2.	▪ Exercise 3.2 ▪ There are 10 questions about the characteristics of function	▪ No contextual question to solve real problem. ▪ There is only 1 question asking for ***the exploration of relation from A to B, and then from B to A***.	
3.	▪ Exercise 3.3 ▪ There are 15 questions of function presentation	▪ There is only 1 contextual question about real problem solving. ▪ There is only 1 question asking for the exploration of mapping from A to B, and then from B to A.	
4.	▪ Exercise 3.4 ▪ There are 8 questions about one-one correspondence	▪ There is no contextual question about real problem solving. ▪ There is no question asking for the exploration of one-one correspondence from A to B, and then from B to A.	
5.	▪ Competency Test 3 ▪ There are 30 questions	▪ There is only 1 contextual question about real problem solving ▪ There is no question asking for the exploration of relation/ function/ one-one correspondence from A to B, and then from B to A.	
6.	▪ Exercise 3.1 to 3.4 and Competency Test 3 ▪ All questions	▪ There is no exploration of algebraic equation form implicitly (ax+by=c); all are dominated by the form f(x)=ax+b or y = ax+b. ▪ There is no exploration of various graphic representation to distinguish function and non-function	▪ Students did not identify implicit equation, therefore found difficulty in converting the implicit into explicit equation form. ▪ Students only identified the linear and square function graphs.

Epistemological obstacles include: (1) Students' inconsistent definition on function, making them not comprehending the definition of function completely because they have limited context and form in constructing the meaning; (2) students conceive the definition of function incompletely, limited only to all members of domain having one pair exactly in codomain, leading to their error in identifying a representation of function or non-function.

Didactical obstacle is detected from inadequacy and error in the presentation of revised-edition handbook for the 8th graders published by Ministry of Education and Culture, resulting some obstacles in the students, including: (1) the error of non-function example presentation in students' textbook; (2) error of function graph presentation related to continuous or discrete function; (3) the example of discussion of questions and exercise questions in

textbooks emphasizing various essential concepts for students' learning experiences including, among others: (a) contextual question containing the real-world problem solving; (b) diversity of relation and mapping direction; (c) presentation of equation in implicit form; (d) presentation of various linear, square, and non-linear curves to explore the representation of function and non-function graphs.

Another finding shows that some students' concept comprehension has reached an intact concept definition image but has limited context in linear function and square function in accordance with learning experience obtained. The students could comprehend function as interdependence relation between two variables with rule and order in the form of algebraic expression (formula), and it can be expressed in various representations. Because of limited context in linear function and square function, the students encounter epistemological obstacles including the difficulty of detecting function or non-function in any non-linear and non-square graphs. Therefore, the author considers the importance of providing an outline conceptual map in the form of function family tree, before entering into the material of relation. It is important to enable the students to know where they have come at and how far they will go in learning function. Some attempts could be taken to improve the learning of function to enable the students to comprehend it completely: (1) preparing and improving the content of textbook using fraxeology; (2) equipping the material with the conceptual map of function family tree; and (3) developing hypothetical learning trajectory of didactical design to solve learning obstacle.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Conflict of Interest

* The authors have no conflicts of interest to declare relevant to the content of this article.

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Author(s) Information

Ika Wulandari
Indonesia University of Education (Universitas Pendidikan Indonesia), Jl. Dr. Setiabudi, No.229, Isola, Kec. Sukasari, Kota Bandung, Jawa Barat 40154, Indonesia
ORCID iD: https://orcid.org/0000-0002-3584-0726
*Contact e-mail: *ikawulandari@upi.edu*

Sufyani Prabawanto
Indonesia University of Education (Universitas Pendidikan Indonesia), Jl. Dr. Setiabudi, No.229, Isola, Kec. Sukasari, Kota Bandung, Jawa Barat 40154, Indonesia
ORCID iD: https://orcid.org/0000-0003-2872-6535

*Corresponding author(s) contact email

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