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Number Sense: Flexibility in Thinking about Numbers

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Abstract: The aim of this research is to understand the flexibility of students in solving problems related to number sense. Flexibility can be observed through the strategies used by students, whether they employ standard algorithmic procedures or utilize intuitive thinking strategies. This study is a qualitative research with a case study method. The research sample consists of 35 eighth-grade students who have covered the material of integers and have been given a written test. Test results are divided into two categories: students who solve problems using number sense strategies and students who do not use number sense strategies. In-depth interviews were conducted with two representative students who took the written test. The interview results indicate that students who solve problems with number sense strategies understand the concept of number density, while students who do not use number sense strategies do not yet have an understanding of the meaning of numbers. Overall, students tend to use procedural methods to solve the given problems. The results of this research are expected to be used as a basis for designing learning activities that accommodate number sense abilities.

Keywords: Numbers, Number sense, Flexibility in thinking.

Introduction

All humans, from the first year of their lives, have a well-developed intuition about numbers. Even in the early stages of development, humans possess the ability to recognize changes in a small collection, when, without direct knowledge, an object has been removed or added to their collection. This ability is referred to as number sense (Tobias Dantzig, 1967). Number sense involves rational, creative, effective, and flexible thinking, allowing students with adequate number sense to solve problems effectively, creatively, analytically, and flexibly. Number sense is built on meaningful learning experiences and not on the use of standard algorithms, such as complex formulas and written calculations. Furthermore, NCTM (1989) states that number sense is an essential skill to be developed in the mathematical process.

In the mathematical assessment dimension within the National Assessment of Educational Progress document (Nelson et al., 2003), number sense is considered a content strand implying that almost all mathematical topics always involve numbers. Someone with good number sense does not require written calculations to answer simple arithmetic but can process them mentally. Students' understanding of numbers aims to develop calculation skills so that this understanding can be used as a flexible thinking strategy in everyday life. For example, when students are confronted with the question "is 87 x 0.09 less than 87, slightly less, slightly more, or much more?" To answer this question, a strong sense of numbers is crucial, with the expectation that students can respond that 87 x 0.09 is much less than 87.

The ability of number sense can be examined based on the strategies employed by students in solving problems, whether using number sense strategies or standard procedures (Yang et al., 2004). Students utilize standard procedure strategies when they apply written algorithms to solve problems (Yang et al., 2004). Furthermore, Yang also states that students are considered to use number sense strategies if they can apply one or more of several number sense indicators, such as the basic meaning of numbers, equivalent forms and representations of numbers, benchmarks, estimation, and the relative impact of arithmetic operations.

Indicators of Number Sense

Number sense can be observed by examining an individual's flexibility with numbers. In the context of learning, this flexibility is observed when students apply one or more of the three components of number sense: understanding the meaning of numbers, understanding arithmetic operations, and applying knowledge of the meaning and operations of numbers to computational settings (McIntosh et al., 1992; McIntosh et al., 1997). These three components are further broken down into six indicators, namely:

1) Understanding of the meaning and size of numbers (number concepts)

The first indicator measures the understanding of the base-ten number system (integers, fractions, and decimals), including patterns and place value that convey the meaning or size of a number. For example, 5/6 is a fraction less than one, approaching one due to the relationship between the numerator and denominator. Similarly, 1000 is a large number when referring to the school population but a small number when referring to the city population. Additionally, the first indicator involves relationships and/or comparisons of numbers to standard benchmarks, including comparing the relative size of numbers within a single representational form.

2) Understanding and use of equivalent forms and representations of numbers (multiple representations).

The second indicator measures the ability to identify and/or rephrase numbers to generate equivalent forms. This ability is related to the use of decomposition and recomposition to rephrase numbers to make them more manageable in processing. Additionally, the second indicator also assesses the ability to associate and/or compare numerical magnitudes with physical references (such as a set of items, shaded areas, or positions on a number line), including intersections between various representational forms.

3) Understanding the meaning and effect of operations (effect of operations).

The third indicator measures the understanding of the meaning and effects of an operation, either in general or related to a specific set of numbers. For example, division means breaking numbers into a certain number of equivalent subgroups, or multiplying by a number less than 1 results in a product less than another factor. This includes assessing the reasonableness of results based on the understanding of numbers and operations used.

4) Understanding and use of equivalent expressions (equivalent expressions).

The fourth indicator is used for reevaluation and/or processing computations more efficiently. This includes understanding and applying the properties of arithmetic (commutative, associative, distributive) to simplify expressions and develop solution strategies (for example, using the distributive property to multiply 7 x 52).

5) Computing and counting strategies

The fifth indicator applies various components of numerical understanding as previously explained in the formulation and implementation of solution processes for calculation or computation situations (estimation, mental computation, paper/pencil, calculator). For example, determining whether 29 x 38 is greater or less than 400, or estimating the number of birds in the sky.

6) Measurement benchmarks

The sixth indicator examines the application of various components of numerical understanding, as previously explained, in formulating and implementing solution processes for measurement situations. Additionally, it considers the understanding and use of standard, non-standard, and/or personal benchmarks (for example, a textbook weighing about one kilogram or 5 oranges equaling one pound, or an angle slightly smaller than a right angle, measuring approximately 85 degrees). It also involves attributes of measurement such as mass, length, capacity, volume, time, and angle.

Method

Research Design

This research employs a qualitative research method with a case study approach. A case study is a type of research that focuses on a social phenomenon and requires in-depth investigation into that phenomenon. It involves the use of various data sources to understand a specific phenomenon (Crowe et al., 2011; Feagin et al., 2016; Heale & Twycross, 2018). This study utilizes a case study as the primary method to identify the strategies used by students in solving a problem on the topic of integers by examining the flexibility of their thought processes.

Participants

This research was conducted at one of the Public Junior High Schools in the city of Palembang in October 2023. The sample selection for the study utilized the purposive sampling technique (Etikan, 2016). Purposive sampling is a method of selecting samples based on specific considerations. The participants in this research were 35 eighth-grade students who had previously studied the material on integers in their previous class. The research was divided into three stages, which are explained as follows.

Stage 1

The first stage is the pre-research stage. In this stage, the researcher validates and tests the questions with eighth-grade students who have covered the material on integers to assess the readability of the questions. The students used for the trial are from a different class than the one involved in the actual research.

Stage 2

The second stage is the implementation stage of the research. In this stage, students are given a written test on the topic of integers that accommodates their number sense abilities. The test consists of seven multiple-choice questions and three essay questions, designed to assess students' number sense abilities. The students' responses are evaluated using a scoring rubric created beforehand. A score of 1 is given if students employ number sense strategies, and a score of 0 is given if students use non-number sense strategies.

Stage 3

The third stage is the post-research stage. In this stage, an evaluation of the students' test results is conducted. A score of 1 is given if students use number sense strategies, and a score of 0 is given if students use non-number sense strategies. After scoring each question, the accumulated scores are used to determine the students' overall test scores for number sense abilities. Students are considered to have good number sense abilities if they score ≥ 70.

After the test results are evaluated, a semi-structured interview (Magaldi & Berler, 2020; Schmidt, 2004) is conducted with representatives from each group: the group of students who answered using number sense strategies and the group who answered using non-number sense strategies. Selecting one participant for an in-depth interview from each group is sufficient for this case study since this qualitative research does not aim to generalize to the chosen sample population. In-depth interviews are conducted to confirm students' written responses, and the obtained results are recorded and transcribed. Interview questions focus on their thought patterns, experiences, and perceptions regarding problem-solving approaches. One of the goals of the student interviews is to understand how students interpret numbers, their thinking processes, and their flexibility in thinking.

Data Analysis

Data analysis in this research is conducted with reference to Huberman and Miles (2002). In the qualitative data analysis stage, it consists of three simultaneous activities: data reduction, data presentation, and drawing conclusions. The present figure illustrates the process.

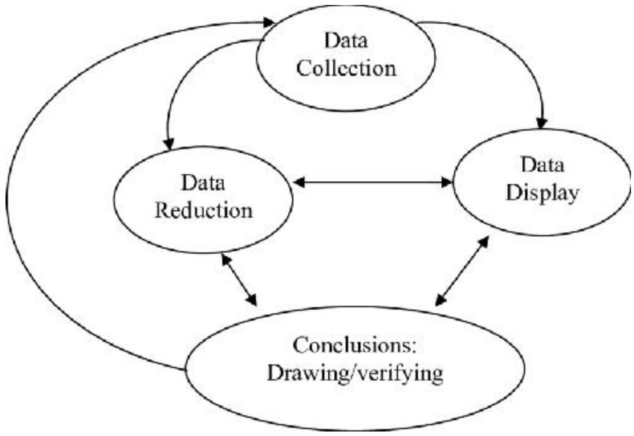


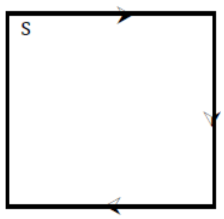
Figure 1. Flowchart of qualitative data analysis

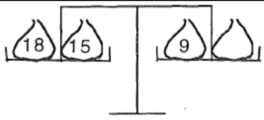
Results and Discussion

Number Sense Test

The results of the author's student test are presented based on number sense indicators and questions representing these indicators. Additionally, students who answered using number sense strategies are also presented and can be seen in the following Table 1.

Table 1. Test result number sense for Indicators

Indicator Number Sense	Question	Students with NS Strategy
(i) Understanding of the meaning and size of numbers (Number Concepts)	➢ How many different fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$. a. None. Why? b. One (Name it: _____) c. A few (Name it: _____) d. Lots (Name it: _____)	P1, P4, P6, P7, P18, P23, P28, P31, P32
(ii) Understanding and use of equivalent forms and representations of numbers (Multiple Representations).	➢ You are going to walk once around a square-shaped field. You start at the corner marked S and move in the direction shown by the arrow. Mark with an X where you will be after $\frac{1}{3}$ of your walk. 	P1, P3, P4, P5, P6, P7, P9, P10, P11, P12, P13, P16, P18, P22, P35
(iii) Understanding the meaning and effect of operations (Effect of Operations).	➢ When a 3-digit number is added to a 3-digit number, the result is … a. Always a 3-digit number b. Always a 4-digit number c. Either a 3 or-digit number d. Either a 3, 4, or 5-digit number	P1, P3, P5, P6, P11, P16, P32, P35
(iv) Understanding and use of equivalent expressions (Equivalent Expressions).	➢ Ayu has balance some bags of marbles. The numbers show how many marbles are in the unmarked bag? Circle your answer.	P1, P2, P3, P4, P5, P6, P7, P11, P12, P13, P14, P15, P16, P17, P19, P20, P21, P22, P24, P25, P26,

			P27, P28, P29, P32, P33, P34, P35
(v)	Computing and Counting Strategies.	➢ About how may days have you lived? Circle your answer. a. 300 c. 30.000 b. 3000 d. 300.000	P1, P3, P6, P7, P9, P10, P13, P14, P17, P23, P24, P25, P30, P31, P32
(vi)	Measurement Benchmarks.	If your heart rate after exercising is 110 beats per minute. How many heartbeats do you feel when you are resting? a. 30 times c. 150 times b. 70 times d. 180 times	P1, P6, P8, P10, P11, P12, P15, P16, P17, P18, P21, P25, P33

The number of participants and the percentage of those who answered the number sense test results can be seen in Figure 2.

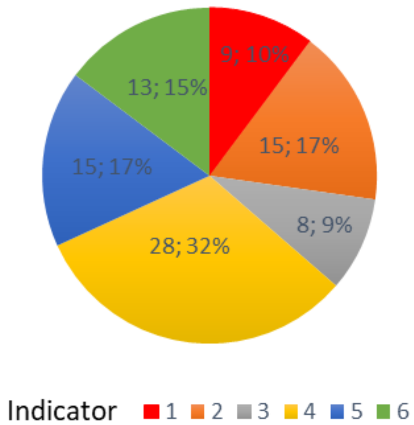


Figure 2. Test result number sense

Understanding of the Meaning and Size of Numbers (Number Concepts)

The first indicator looks at students' understanding of number density. Questions focusing on number density and the concept of "betweenness" are consistently challenging for students. For instance, when given the question "How many different fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$? only 10% of students responded correctly and used number sense strategies to solve it. This result indicates that students' understanding of fraction density is still weak. Below are some of the student responses.

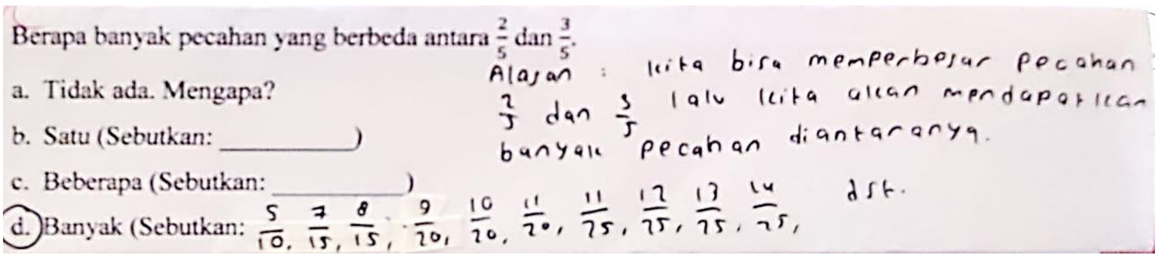


Figure 3. Students' responses to the first indicator with number sense strategies

Students with number sense will answer that there are an infinite number of fractions between the numbers 2/5 and 3/5. In this case, students can comprehend the concept of the density of numbers and are also capable of providing examples of infinite numbers as mentioned in the research (Yang, 2005). Furthermore, students also understand how to operate numbers flexibly, using mental calculations and logical estimations, making predictions, and understanding the numerical relationships among mathematical concepts, facts, and skills (Guedj, 1998; Yang et al., 2008; Reys et al., 1999).

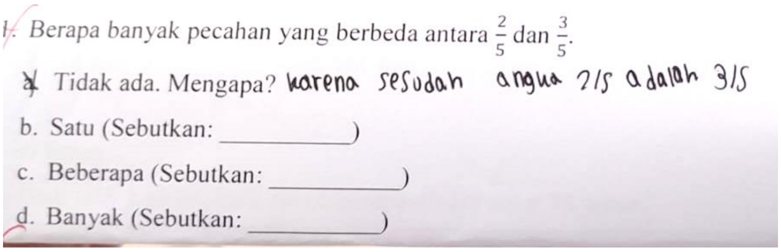


Figure 4. Students' responses to the first indicator with non-number sense strategies

Students who answer with non-number sense strategies do not recognize the infinite number of different fractions between $\frac{2}{5}$ and $\frac{3}{5}$, and they may not have an understanding of the meaning of numbers, as evidenced by responses that state there are no fractions between $\frac{2}{5}$ and $\frac{3}{5}$ namely $\frac{5}{10}$.

Understanding and Use of Equivalent Forms and Representations of Numbers (Multiple Representations)

The question representing the second indicator representing a position one-third of the way around a square as represented in second question was difficult for most students. Incorrect responses to this question highlight the lack of a benchmark with which to compare and make judgments about $\frac{1}{3}$. In designing the question, researcher felt that students who used each side of the square to represent $\frac{1}{4}$ would only need to be able to understand the relationship between $\frac{1}{4}$ and $\frac{1}{3}$ to respond correctly to the question. An alternative approach would be to think about the line making up the entire square as a "smooth" one and then to mentally section off the line into three equal segments. In fact, many of the incorrect responses related to using the vertices as "thirds" markers. In other words, think of standing at one comer of the square. What you see from this vantage point is three comers and each might be viewed (incorrectly) as representing a third of the square. Other students reasoned that the mark should go beyond the first corner but not as far as the second (halfway) comer. They then concluded that the $\frac{1}{3}$ mark should go halfway down the second side (be halfway between $\frac{1}{4}$ and $\frac{1}{2}$).

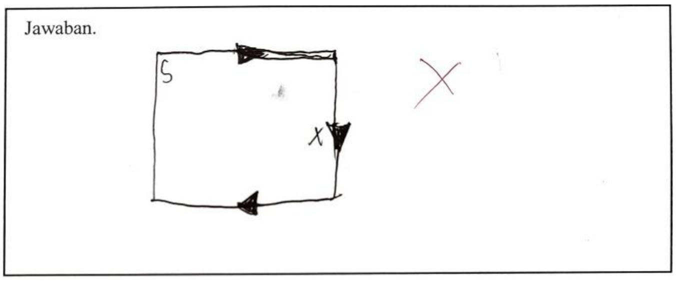


Figure 4. Students' responses to the second indicator

Understanding the Meaning and Effect of Operations (Effect of Operations)

Questions measuring the third indicator required the correct response to recognize that the sum of two 3-digit numbers would be either a 3- or a 4-digit number. The percentages of correct responses are quite low, suggesting that despite the numerous addition computation exercises undertaken by these students, they do not seem to have a sound concept of what occurs when two numbers are added. It would have been interesting to observe how the students handled this item.

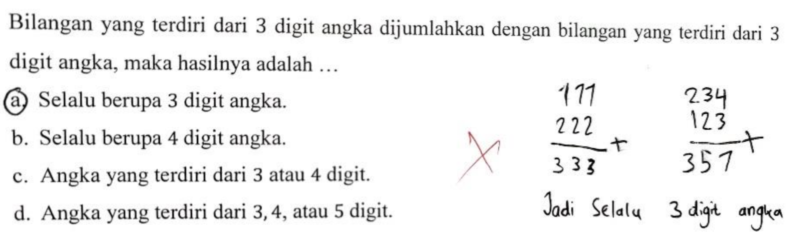


Figure 5. Students' responses to the third indicator

Understanding and Use of Equivalent Expressions (Equivalent Expressions)

Students in Grade VIII of junior high school, who have studied integers, were given a diagram with a beam balance showing two bags containing 18 and 15 marbles respectively, balancing two other bags - one with 15 marbles and the other with an unknown number to be identified. The percentage of students who correctly identified the number in the unlabelled bag was 28 students. This result indicates that there are still students who do not seem to understand the concept of commutativity in the addition of whole numbers.

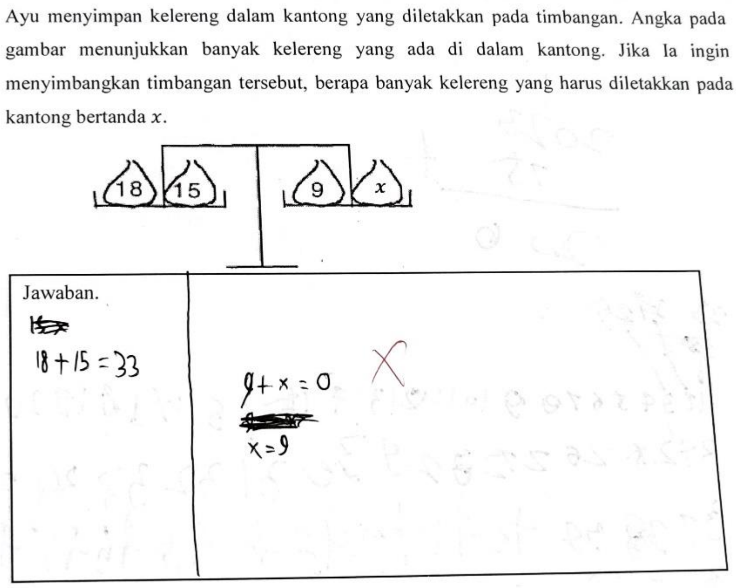


Figure 6 Students' Responses to the fourth indicator

Computing and Counting Strategies

The next question students are asked to estimate how many days they had lived from four given choices. The results show that students had great difficulty in distinguishing between 300, 3000, 30 000, and 300 000 days as their approximate age. Obviously this problem is complicated for the students who are used to thinking of their age in years. However, performances again indicate a poor grasp of larger numbers among students.

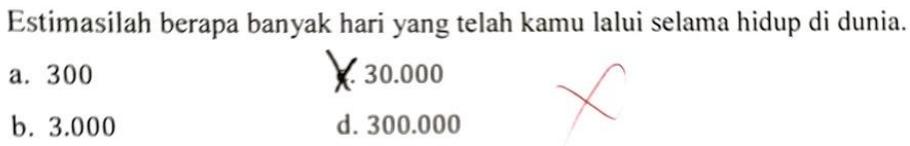


Figure 7. Students' responses to the fifth indicator

Measurement Benchmarks

In this question, students are asked to estimate the number of heartbeats after exercising and during rest. In fact, many students still do not understand the standard units of human heart rate. So when asked about the number of heartbeats during rest, students have difficulty determining whether the number of heartbeats will increase or decrease compared to after exercising.

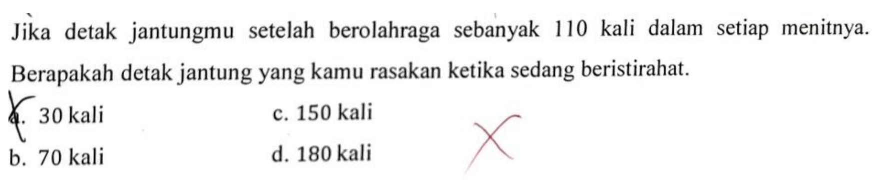


Figure 8. Students' responses to the sixth indicator

Interview Results

Based on the written test results provided above, the author selected one student from each category. The first student (S1) is the one who answered the questions using number sense strategies, while the second student (S2) is the one who employed non-number sense strategies in their responses. A number sense strategy is defined as the ability to utilize one or more indicators, such as the fundamental meanings of numbers, equivalent forms, and numerical representations (Yang et al., 2004). Consequently, the author conducted in-depth interviews with the students, focusing on indicators that gauge conceptual understanding and numerical proficiency. The interviewer is identified as I. The following excerpt from the interview transcript pertains to the flexibility of students' thinking.

The first student,

I : According to S1, are there numbers between 2 and 3?
S1 : Yes
I : Can you mention them?
S1 : 2.1, 2.2, 2.3, and so on
I : How about between 2.1 and 2.2, are there numbers?
S1 : Hmm... (appears to be thinking). I think there are.
I : Please mention them
S1 : 2.11, is that correct?
I : It could be. Why did you come up with the number 2.11?
S1 : (student is silent for a moment, there is a delay of about 5 seconds) ... I just added numbers behind the decimal point.

Then, the interviewer shifted to questions involving fractional numbers to further explore information related to S1's answer to question number 1.

I : If there are other numbers between $\frac{2}{5}$ and $\frac{3}{5}$?
S1 : Is this the question you gave yesterday, ma'am? (the student smiles)
I : Yes, I want to know why you chose D (D is the selected answer).
S1 : I enlarged the denominator, ma'am. Initially, I made it 10, then there's $\frac{5}{10}$. Then I multiplied the denominator to 15, 20, and so on. It turns out there are many numbers between $\frac{2}{5}$ and $\frac{3}{5}$

From the interview results, it is evident that student S1 can mention many numbers between $\frac{2}{5}$ and $\frac{3}{5}$, The student is already able to apply their understanding of the base-ten number system and formulate different fractions to generate equivalent forms, making them more manageable in processing.

The second student,

I : According to S2, are there numbers between 2 and 3?
S2 : No, ma'am, it's already 2, approaching 3
I : Are you sure there are none? Is 2.1 a number or not?
S2 : (pauses for a moment) ... yes, ma'am, that's also a number. I thought the answer shouldn't have a decimal point.
I : It's allowed as long as it's a number. How about between 2.1 and 2.2, are there numbers?
S2 : (pauses for a moment) ... just add one number behind it, right, ma'am? ... (the student seems unsure and seems to follow a previous pattern, not based on knowledge)
I : Maybe yes.
S2 : (remains silent)
I : For question number 1, why did you choose answer A?
S2 : because the denominator is the same, ma'am, and the number 2 is almost 3. But can we add a decimal point like before?
I : It's possible.
S2 : I'm confused, ma'am, it's a bit strange because it's not usually like that. We have never been given such a question before.

From the interview results, it is evident that student S2 has not yet grasped the concept of the density of numbers. The student seems to be memorizing question patterns and answering without relying on their knowledge of numbers. Most students do not understand the meaning of numbers and are often taught routine problems that do not encourage flexible thinking.

Conclusion

This study found that many students still lack flexible thinking in solving problems. On average, students tend to use standard procedures often taught by teachers. Additionally, classroom learning inadequately accommodates the development of number sense skills.

Recommendations

A suggestion for further research is to develop a learning design that focuses on instilling concepts and understanding of numbers because all mathematical topics always involve numbers. Furthermore, understanding numbers aims to develop computational skills that can be used as thinking strategies in everyday life. The learning design can consider mathematical knowledge from scholarly knowledge so that the constructed concepts can be justified for their accuracy.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Acknowledgements or Notes

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