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Development of Manipulative: Understanding the Concept of Integer Addition and Subtraction

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Abstract: Integers are often considered complex material that students struggle to grasp. Operations involving the addition and subtraction of whole numbers pose significant challenges for elementary students, who may encounter difficulties such as understanding the concept of negative numbers, subtracting larger numbers from smaller ones, and performing addition involving both negative and positive numbers. These challenges highlight the need for teachers to devise effective strategies and models that support students in learning integers. Providing engaging and interactive learning experiences could be an optimal approach to help students reason about integers. Consequently, the use of media or tools may be necessary to make the concept of integers more concrete and less abstract. This research aims to develop a valid, practical, and effective manipulative using the Thiagarajan 4D model, which consists of defining, designing, developing, and disseminating phases. The study involved eleven sixth-grade elementary school students as participants. A pretest and posttest on integer addition and subtraction were administered to evaluate the differences in students' abilities before and after using the product. The results suggest that students found it easier to conceptualize integers as assets and debts with the aid of the manipulatives. The final product, a manipulative tool, is named the 'Integer Operations Board.

Keywords: Concept of integer, Subtraction, Addition, Manipulative

Introduction

One of the key topics covered in grade 6 is integer operations, which serves as a foundation for more advanced concepts, such as algebraic operations (Faznur et al., 2020). Students are introduced to various types of integers—positive, zero, and negative. Research has shown that understanding addition and subtraction of integers remains a challenge for many students. Baroody (1999) found that children do not fully grasp the relationship between addition and subtraction of whole numbers. Bofferding (2014) examined students' understanding of negative numbers and found that students tend to perceive negative numbers similarly to positive numbers, assuming that the larger the number, the greater its value. For instance, they may interpret –5 as greater than –3. Sidik et al. (2021) also found that many students struggle to understand the questions, face difficulties with the order of numbers in operations due to the presence of positive and negative signs, and often make mistakes in determining the results of integer addition or subtraction. Based on these facts it can be concluded that students still struggle with addition and subtraction while learning integer numbers, particularly when dealing with negative values.

Minus Sign

Students' difficulties in solving symbolic addition and subtraction of integers are caused by their lack of understanding of negative numbers (Kumar et al., 2017; Stephan & Akyuz, 2012). Negative numbers often become a problem because previously students only recognized the negative sign as a subtraction operation, but now the negative sign is also introduced as a sign attached to integers. For example, in the operation 2 – (–1),

students find it difficult to distinguish between the negative sign as an operation and the negative sign as a sign of an integer (Cetin, 2019). Vlassis (2008) explained that the negative sign can be used in three ways: as a unary, binary, and symmetric function as follow:

Unary Function

The negative sign serves as a label that indicates a negative integer or a sign attached to the integer, such as –2 and –3.

Binary Function

The negative sign serves as the sign of the subtraction operation, as seen in the operation 7 – 2 (if you have 7 candies and give 2 away, 5 are left).

Symmetric Function

The negative sign represents the opposite of another integer, for example – (–5), where the first negative sign serves to change – (–5) into +5, creating a positive value. Let's examine the operation 2 – (–3), or "2 minus negative 3." In this case, subtracting a negative number is equivalent to adding its opposite, so 2 – (–3) can be rewritten as 2 + 3, resulting in 5. By interpreting integers through the concept of symmetry, we see that when an operation involves the subtraction of a negative number, such as 2 – (–3), the result matches the addition operation 2 + 3, which equals 5.

Another challenge arises when students try to subtract a larger number from a smaller number or add a negative number to a positive one. This also becomes a problem for students because the concept of a negative sign is not straightforward to illustrate in everyday context. For example, with a simple operation like 5 – 3, a teacher might explain that if a student has 5 candies and gives 3 away, they'll have 2 candies left—an easy scenario to visualize. However, with the operation 3 – 5, it's challenging to illustrate how a student could give away 5 candies when they only have 3. This lack of a concrete, relatable example makes it harder for students to understand operations involving negatives.

Models and Contexts

In exploring representations to help students gain a deeper understanding of the concept of integers, teachers should consider the roles of models and contexts (Stephan & Akyuz, 2012). Through modeling, teachers can introduce students to properties such as commutativity in addition and the concept of neutralization. Regarding the commutative property of addition, this means that when two numbers are added, the order of those numbers does not affect the sum. This property holds true even when the numbers being added have different signs. For instance, – 2 + 1 is the same as 1+ (– 2). This reinforces the idea that addition is flexible and can help students understand integer operations better.

For the neutralization model, teachers can explain that neutralization can be defined as a zero-valued quantity. In this model, positive and negative numbers are said to cancel each other out; thus, a set of numbers is considered neutral when the quantities of positive and negative numbers are equal. This concept can be effectively illustrated using the context of money and debt. For instance, if an individual has a debt of Rp10,000 but also has money worth Rp10,000, they can fully settle their debt through this cancellation. In symbolic terms, this relationship can be expressed as – 10.000 + 10.000 = 0 (Stephan & Akyuz, 2012).

Next, let's consider context-oriented representations. Some effective contexts for connecting integer operations to everyday life include temperature, altitudes, financial situations like money and debt, profit and loss (Kumar et al., 2017). By utilizing these relatable examples, students can grasp the concept of integers more clearly and develop greater cognitive flexibility. This flexibility is an essential aspect often overlooked by teachers. Lamb et al. (2023) argue that an emphasis on flexibility in instruction not only boosts flexibility itself but also enhances overall performance.

Manipulative as a Tool

To foster flexible thinking, teachers must be ready to provide meaningful learning in the simplest and most enjoyable way. Bosse et al. (2016) mentioned that one of the best ways to reduce the abstraction of mathematics is with the help of manipulatives. Ardina et al. (2019) define manipulatives as concrete objects that students can touch, move, and relocate. The use of manipulatives is highly recommended in mathematics learning because they can bridge the gap between the teacher's intentions and the students more easily (Annisa et al., 2022; Purnawati & Qohar, 2022). Other advantages of manipulatives include enhancing students' learning motivation and self-confidence, actively involving students, and fostering students' enthusiasm for learning mathematics (Anjani et al., 2021; Capuno et al., 2019; Murni et al., 2022; Wardana et al., 2022).

Several studies have demonstrated the use of manipulatives to enhance students' understanding of integers. For instance, Bolyard & Moyer-Packenham (2012) utilized virtual manipulatives, such as a virtual number line applet and a virtual integer chips applet, to illustrate the addition and subtraction of integers. Similarly, Cetin (2019) explained integer operations through the opposite model, employing ordered pairs to represent their integer equivalents. For example, the number –2 can be represented as (0, 2), (1, 3), (2, 4), and (3, 5), where the first component of these ordered pairs signifies positive values, while the second represents negative ones. Additionally, Sahat et al. (2018) employed manipulatives in the form of two-colored Lego stacks to differentiate between positive and negative numbers. A vertical stack of Legos indicates the tens place. This research highlights the widespread application of manipulatives in supporting students' comprehension of integers. Furthermore, Listrianti et al. (2022) used playing cards to enhance students' abilities in performing integer addition operations at a madrasah.

In implementing manipulative tools, it is crucial to adapt their creation to the specific material being taught and the unique needs of the students (Suryawan et al., 2023). When it comes to the topic of integers, students frequently struggle to differentiate the roles of positive and negative numbers (Mustika et al., 2023). To address this challenge, the researchers aim to develop manipulative media that incorporate models and contexts designed to help students grasp the different functions of positive and negative signs and to facilitate their understanding of addition and subtraction operations involving integers.

The manipulative being developed is based on the chip model concept, which has been widely utilized in previous studies (Bolyard & Moyer-Packenham, 2012; Bozkurt et al., 2022; Murray, 2018). Previous research on the chip model primarily focused on virtual forms and did not include accompanying worksheets. In this development, the researcher plans to create a physical chip model manipulative that is easy to hold and manipulate. Furthermore, the researcher will provide worksheets to guide students in exploring the roles of positive and negative signs in integer operations. Based on the background explained above, the researcher aims to develop a valid, practical, and effective manipulative media known as Integer Operations Board.

Method

This research uses 4D model by Thiagarajan which has four stages: define is to analyze the development needs, design is to create the product, develop is to validate the product, and disseminate to publish the final product. The subjects in this study are eleven sixth-grade elementary school students. Prior to this research, all participants had already learned about the addition and subtraction of integers from their regular teacher. This criterion was selected because the researcher aims to assess whether the developed manipulative can effectively enhance students' understanding of the concepts of integer addition and subtraction.

The instruments used in this study include the integer operation board as manipulative, a student response questionnaire, and both pretest and posttest assessments. During the development stage, several activities will be conducted, including product validation to determine the validity of the product, product trials to evaluate the product, pretest and posttest to measure the effectiveness of the product, and a response questionnaire will be administered to evaluate the practicality of the product.

Firstly, to validate the product, a media validation questionnaire will be utilized. Each validator will evaluate the materials and provide constructive feedback and suggestions prior to the product's implementation. The validation team comprises three experts: one professor of mathematics education and two graduate students specializing in mathematics education. Secondly, the student response questionnaire is designed to assess the practicality of the learning media from the perspective of the users, i.e., the students. Finally, the pretest and posttest assessments aim to measure the students' understanding of integer operations both before and after the

introduction of the learning media. Below is a Likert scale table that will be used to evaluate the validity of the media and the responses from the students:

Table 1. Likert scale criteria

Score	**Description**
4	Strongly agree with the statement
3	Agree with the statement
2	Somewhat agree with the statement
1	Disagree with the statement

Validity Analysis

The validity of the media is calculated based on the average score for each validity criterion. The total average score from each validator is then determined. Below is the table showing the validity levels of the media:

Table 2. Media Validity Levels

Average Validity Score (V)	Validity Levels
$1 \leq V < 2$	Not valid
$2 \leq V < 3$	Less valid
$3 \leq V < 4$	Valid
$V = 4$	Highly valid

Practicality Analysis

The practicality of the media is calculated by averaging the scores for each practicality criterion, followed by the total average score from each student. Below is the table showing the practicality levels of the media:

Table 3. Media practicality levels

Average Practicality Score (P)	Practicality Levels
$1 \leq Pr < 2$	Not practical
$2 \leq Pr < 3$	Less practical
$3 \leq Pr < 4$	Practical
$Pr = 4$	Highly practical

Effectiveness Analysis

The effectiveness of the media is assessed using the N-Gain score, which is derived from the pretest and posttest scores. Below are the N-Gain calculations and their corresponding criteria:

Table 4. Effectiveness Level

N-Gain Value	N-Gain criteria
$g > 0{,}70$	High
$0{,}30 \leq g \leq 0{,}70$	Medium
$g < 0{,}30$	Low

Result and Discussion

Result

The result of this research is a valid, practical, and effective integer operation board. Here are the results of the media development, which includes four stages: define, design, develop, and disseminate.

Define

This stage involves curriculum analysis, student analysis, concept analysis, and the establishment of learning objectives tailored to the needs of the media. During the curriculum analysis, it was determined that the focus for 6th-grade mathematics is on the operations of addition and subtraction of integers, excluding multiplication and division. In the student analysis, it was identified that 6th-grade students often struggle with understanding negative integers. Many students have difficulty distinguishing between the negative sign as an indicator of subtraction and the negative sign as a representation of a number. Additionally, they face challenges with addition when dealing with negative numbers and with subtraction when the number being subtracted is smaller than the initial number. To facilitate students' understanding of addition and subtraction, the researcher documented every potential operation that might arise in integer operations, as follows:

Addition	Subtraction
positive number + positive number	positive number – negative number
positive number + negative number	positive number – positive number
negative number + positive number	negative number – negative number
negative number + negative number	negative number – positive number

These four types of operations will be included in the student worksheets. Interestingly, operations in the same row will yield the same results. These results will later help lay the groundwork for students to draw flexible conclusions regarding the addition and subtraction of integers. Additionally, the concept analysis revealed that the teacher does not utilize any media as a supportive learning tool and fails to introduce students to various types of representations (such as symbolic, visual, and verbal contexts) to explain integer operations. Based on these analytical activities, the researcher has established clear learning objectives: to help students grasp the concept of negative numbers and to enable them to solve integer operations (both addition and subtraction) using the integer operation board.

Design

At this stage, the researcher begins planning the design, concept, tools, and materials needed to create the integer operation board. A simple design was chosen to ensure that integer calculations are clearly visible. The selected tools and materials are not only easy to find but also durable, ensuring that the board can be used repeatedly.

The integer operation board is made using a white magnetic board measuring 20 cm × 30 cm. The other components include blue magnetic pins to represent positive numbers and red magnetic pins to represent negative numbers. These pins serve as counting tools that students can manipulate independently. Each blue pin represents +1, while each red pin represents –1. The selected magnetic board can also be used as a base for writing answers or students' doodles as they work on understanding the problem presented in the worksheet.

Figure 1. Integer operation board

Develop

This stage includes product validation, product trials, pretest and posttest assessments, and filling out student response questionnaires. Each of the activities mentioned above is explained in detail below:

Table 5. Media validation results

Criteria	Average scores for each criterion
Learning Media Content	
The learning media helps students build an understanding of mathematical concepts	3,33
The activities provided encourage positive interaction between students and the learning media	3,66
The activities within the use of the learning media align with the learning objectives	3,66
The learning media does not contain ambiguity	3,33
Design and Visual Appeal of Manipulative Media	
The learning media is visually appealing	3
The size is proportional	3,66
Benefits of the Learning Media	
The media attract students' interest in learning mathematics	3,66
The media encourages students to be more active	3,66
The media fosters teamwork in groups	3,33
Validity Score	3,47

The analysis revealed that the validity score of the media is 3.47, indicating that the integer operation board is valid and suitable for implementation in learning. During the validation process, the validators recommended to supplement this product with student worksheets to facilitate independent practice and to allow students to engage with problems according to the guidelines provided.

Subsequently, a trial was conducted with all sixth-grade students who had already covered the concepts of addition and subtraction of integers. This trial aimed to evaluate students' abilities before and after using the integer operation board. Initially, a pretest was administered as a baseline assessment to measure students' understanding of integer operations. Following the pretest, students participated in lessons focused on addition and subtraction, employing the integer operation board as a learning tool. Each student received a worksheet designed with practice problems that required the use of the board for resolution.

The learning sessions using the integer operation board were conducted over two meetings over two days. Once all activities outlined in the student worksheets were completed—including discussions, presentations, and question-and-answer sessions—students undertook a posttest to assess improvements in their problem-solving skills. The pretest and posttest were used to analyze the effectiveness of the media by calculating the N-gain from both tests. The pretest and posttest data were analyzed to evaluate the effectiveness of the media by calculating the N-gain for both assessments. The resulting N-gain of 0.76 signifies a high level of effectiveness, demonstrating that the integer operation board significantly enhances students' abilities in performing addition and subtraction of integers. On the final day of the study, students were administered a questionnaire to evaluate their experience with the media. The results are as follows:

Table 6. Media practicality results

Criteria	Average scores for each criterion
Material Presentation	
The mathematics learning media is easy for me to use	3,64
The presentation of problems in the mathematics learning media helps me understand the concept of integer operations	4,00
I enjoy learning mathematics through this learning media because it is interesting	3,55
This learning media makes me like mathematics	3,00
This learning media makes me actively engage in learning mathematics	2,73
This learning media makes me want to explore mathematics further	3,00
Language and Appearance	
The learning media is visually appealing	3,91
The instructions and information presented are easy for me to understand	3,64
Practicality Score	3,43

Based on the analysis, the practicality score of the media was found to be 3.43, indicating that the integer operation board is practical for use. The highest average score was recorded for the second indicator, which states that the presentation of problems on the integer operation board aids students in understanding the concepts of integer operations. This outcome further suggests that the integer operation board received positive feedback and effectively enhanced students' abilities to solve addition and subtraction problems involving integers.

Disseminate

The dissemination phase occurs after the media has met the criteria for validity, practicality, and effectiveness. In this stage, the integer operation board is provided to the subjects' school. This dissemination aims to facilitate the implementation of the media in supporting the learning of addition and subtraction of integers, as well as to serve as a field evaluation tool to assess the product's effectiveness and determine if further improvements are necessary.

Discussion

The ability of 6th-grade students to solve integer operation problems is still quite limited. Many students struggle to understand the concepts of positive and negative numbers, leading to difficulties in performing addition and subtraction. During the initial phase of the trial, the researcher took on the role of teacher, starting by prompting students to recall their understanding of integers, positive numbers, zero, and negative numbers. However, the students' responses were not satisfactory, indicating a lack of a solid understanding of these concepts.

To address this gap, the researcher introduced relatable contexts, specifically using money and debt as examples. This approach proved effective, as 6th-grade students often find it easier to comprehend integers when positive numbers are associated with money, negative numbers are linked to debt, and zero represents a balance where money and debt are equal. This contextualization helps clarify the abstract concepts of integers for the students, facilitating their understanding.

For instance, the operation 5 + (–2) can be verbalized as, “If you have 5 rupiah and you owe 2 rupiah, how much money do you have left?” Students can easily respond with 3. However, challenges arise with the problem 3 – (–4). Students find it difficult to verbalize this operation due to the presence of two negative signs, which can confuse their concentration. This situation clearly calls for an appropriate model. Hence, the integer operation board serves as a tool to represent positive and negative numbers more concretely, assisting students in grasping concepts.

On the integer operation board, students are provided with blue and red magnetic pins that can be attached to the board. Each blue pin represents the positive integer 1, while each red pin represents the negative integer –1. This color distinction helps students easily identify the types of integers they will be working with. Before proceeding to addition and subtraction operations, the teacher explains that addition means adding pins to the board, while subtraction means removing pins from the board. Additionally, the teacher introduces the concept of neutralization (Stephan & Akyuz, 2012), explaining that a pair of one blue pin and one red pin equates to zero, also known as zero pairs.

For example, in the first problem, 5 + (–2) can be represented by placing 5 blue pins (positive) and 2 red pins (negative) on the board. Once all the pins are attached, students can observe that there are two zero pairs, and the remaining pins (which are not part of any zero pair) represent the answer. In this case, there are 3 blue pins left, indicating that the result is 3. Through this activity, students successfully learn to add positive and negative integers with ease.

For the second problem, 3 – (–4) can be represented by placing 3 blue pins, while students must take away 4 red pins. The challenge arises when students realize they only have 3 blue pins, then how can they take away 4 red pins from the board? To address this issue, students are reminded of the neutralization model, which involves adding zero pairs to the board until they reach the required number of pins for the operation. Since they need to take away 4 red pins, they add four zero pairs (which consist of 4 blue pins and 4 red pins) to the board. Now, students have 3 original blue pins, 4 new blue pins, and 4 new red pins. When they take away the 4 red pins, they find that 7 blue pins remain (3 original blue pins plus 4 new blue pins). This exercise enables students to

recognize that certain operations yield the same result even when different types of numbers are used. For example,
3 – (–4) yields the same result as 3 + 4.

These kind of operations can also be explained through the concept of symmetry, where the inverse of three minus negative four is equivalent to three plus four. This symmetry function applies to subtraction problems, especially when a smaller number subtracted from a larger one. For example, in the case of 5 – 8, the symmetrical form can be expressed as 5 + (–8), which can be verbalized as, "You have five units of money, and then you owe eight; thus, your remaining debt is three," or symbolically written as (– 3).

When solving this on the integer operation board, the first step is to place five blue pins (representing positive values) on the board. Next, students need to take away eight blue pins. Since there are only five blue pins on the board, they can use the concept of neutralization by adding three zero pairs (which consist of three blue pins and three red pins) to the board to facilitate the operation of removing eight blue pins. Now, they have five blue pins, three additional blue pins, and three red pins. After taking away the eight blue pins, they will find that three red pins remain. Therefore, the result of 5 – 8 is –3. This exercise helps students overcome difficulties when encountering subtraction problems where the number being subtracted is larger.

After being introduced to the model and context described above, all students began working according to the sequence outlined in the student worksheets. In this worksheet, students were tasked with completing the following addition and subtraction operations:

Addition	Subtraction
3 + 4 =	3 – (– 4) =
3 + (– 4) =	3 – 4 =
– 3 + 4 =	– 3 – (– 4) =
– 3 + (– 4) =	– 3 – 4 =

If we observe closely, the addition and subtraction problems in the first row yield the same value. Thus, students were challenged to calculate all these tasks using the integer operation board, then compare whether the result in every row is equal. This comparative activity is useful in demonstrating to students that when two signs are consecutive, they can be simplified using the concept of symmetry.

After comparing the results of the operations above, many questions arose from the students regarding the roles of negative and positive signs. For instance, they wondered if the presence of a positive sign (+) next to a negative sign (–) indicates a subtraction operation, or if two negative signs (–) together imply addition. These inquiries suggest that students are beginning to grasp the roles of positive and negative signs and are validating the concept of symmetry through the board.

Next, students addressed the addition of positive numbers with negative numbers. In the case of addition, if the negative number is larger, the result will be negative. Similarly, for subtraction, if a smaller number is subtracted from a larger number, the outcome will also be negative. For example, in the addition of – 4 + 3 and the subtraction 3 – 4, both operations will yield the negative number, which is –1.

A possible question that may arise next is why the values of these two different operations can be the same. In this case, the numbers being operated on are – 4 dan 3. Although one operation involves addition and the other subtraction, the results can be the same due to the commutative property of addition, which states that – 4 + 3 = 3 + (– 4). Based on this, it is clear that 3 + (– 4) is equivalent to 3 – 4, meaning the results of both operations will indeed be the same. However, what about the operation 3 – 4? Is it the same as 4 – 3? The answer is no. The reason is that 3 – 4 = 3 + (– 4) but 4 – 3 = 4 + (– 3) which is 3 + (– 4) is not the same as 4 + (– 3). Therefore, it can be concluded that subtraction does not possess the commutative property.

Based on the activities outlined above, the integer operation board significantly enhances student engagement in learning about integer operations. Additionally, students actively asked questions, engaged in discussions with peers, and successfully presented their practice results related to integers in front of the class. The interactions among students fostered a conducive learning environment (Jumrawarsi & Suhaili, 2020), leading to increased focus and full attention during the learning process. According to research by Wulandari & Qohar (2022), active participation in the learning process effectively helps students avoid boredom, which can adversely affect their learning persistence. The more students enjoy the learning experience, the easier it is for them to grasp the material concepts.

The high scores obtained from the response questionnaire also indicate that students were very enthusiastic about learning with the integer operation board. As explained by Huda & Qohar (2021), using manipulative media in learning can enhance student engagement in solving problems, both individually and in groups. This undoubtedly has a positive impact on students' abilities regarding addition and subtraction of integers.

The integer operation board is designed to assist students in understanding the role of negative numbers, performing operations on two integers with different signs, and facilitating teachers in modeling addition and subtraction operations (such as neutralization and symmetric function). Rahmawati & Roesdiana (2022) state that the essence of mastering mathematics lies in understanding its concepts. Such concepts are often overlooked by teachers, leading to difficulties for students when encountering different types of problems. Therefore, the integer operation board serves as an effective manipulative to provide an engaging and enjoyable learning experience, helping students to better understand the concepts of integer operations.

Conclusion

Teaching integer concepts to 6th graders can be quite challenging. However, the introduction of the integer operation board makes it easier for students to grasp the role of negative numbers, learn how to subtract a smaller number from a larger one, and understand how to add both positive and negative numbers. The use of representations, including models and real-life contexts, plays a crucial role in helping students comprehend addition and subtraction concepts. Models allow students to visualize integer operations as tangible activities, while contextual examples enable teachers to introduce essential concepts like negative signs, symmetry functions, the property of neutralization, and the commutative property in integer operations (Stephan & Akyuz, 2012).

The manipulatives developed, along with the provided worksheets, significantly enhance students' interactive practice opportunities. Although the use of model chips as interactive tools may not have been widely explored in previous research, they have proven effective in facilitating a deeper understanding of addition and subtraction. Additionally, manipulatives support the overall learning process and positively influence students' skills in performing addition and subtraction with whole numbers (Bolyard & Moyer-Packenham, 2012; Bosse et al., 2016; Kwakye & Aggrey, 2022; Listrianti et al., 2022; Sahat et al., 2018; Yuliwijayanti et al., 2021).

Based on the findings from the trials and subsequent analysis, it can be concluded that the integer operation board has successfully met the validity criteria, achieving a validity score of 3.47. Additionally, it demonstrated practicality with a score of 3.43, and its effectiveness is evidenced by the pretest and posttest results, which yielded an N-Gain of 0.76. These outcomes indicate that the integer operation board effectively supports the learning of integer operations in 6th grade. Moreover, it has been shown to enhance students' motivation and confidence, actively engage them in the learning process, and nurture their enthusiasm for mathematics.

Recommendation

Students' abilities and interests can vary significantly across different contexts. Therefore, teachers play a crucial role in creating optimal learning experiences. It’s important for teachers to experiment with various representations to better understand their students' characteristics and how effectively they grasp the material presented in different ways. This consideration is vital because the language and style used by the teacher can greatly influence students' thinking processes (Bofferding, 2010). Based on the findings of this study, it is recommended that teachers introduce concepts such as negative signs, the property of neutralization, symmetry functions, and the commutative property when teaching integer-related topics (Isik, 2018). For future research, it would be beneficial for teachers to develop manipulative media that can assist students in solving multiplication and division operations involving integers.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPESS Journal belongs to the author.

Acknowledgements or Notes

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