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A Proposed Professional Development Program Based on Algebraic Habits of Mind

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Abstract: This study, which presents a systematic review conducted within the TÜBİTAK 1001 Project (No. 223K317), aims to propose a professional development program based on the Algebraic Habits of Mind (AHoM) framework. This program is designed to develop the algebraic thinking skills of pre-service and in-service teachers. It consists of eight modules, each addressing a specific theme, learning outcomes, content, instructional activities, and assessment processes through a holistic approach. The fundamental goal is for teachers to conceptualize algebraic thinking not merely as an operational domain, but as one integrated with higher-level cognitive processes such as generalizing, justifying, symbolizing, and establishing relationships between multiple representations. In this context, special emphasis has been placed on the teachers' guidance roles and strategies for intervening in students' thinking processes. The study aims to contribute to the algebra teaching literature and serve as a model for the design of professional development programs development programs.

Keywords: Algebraic thinking, Algebraic habits of mind, Professional development, Teacher education, Algebra teaching

Introduction

Algebra, one of the core areas of mathematics education, encompasses not only operations with symbols but also higher-level thinking skills such as abstraction, generalization, and representing relationships. Therefore, developing students' algebraic thinking skills from an early age is critically important for the deepening of mathematical understanding. However, research in Turkey indicates that pre-service and in-service teachers predominantly emphasize procedural approaches in algebra teaching, struggling with making sense of symbol use and generalizing. This situation leads students to perceive algebra merely as a collection of mechanical operations, hindering the adequate development of conceptual understanding.

At this point, the Algebraic Habits of Mind (AHoM) framework provides a strong theoretical foundation for teachers to support students' algebraic thinking processes. AHoM guides students in conceptualizing algebraic concepts and generalizing through three fundamental dimensions that deepen mathematical thinking: (doing/undoing, building functional relationships, and abstracting from computation). Professional development for teachers in using this framework will strengthen conceptual algebra instruction in classroom practices and deepen students' mathematical understanding.

In this context, the study proposes an AHoM-based professional development program. The program aims to enhance teachers' skills in supporting algebraic thinking processes and to equip them with pedagogical strategies that will make students' mental algebraic habits visible in the classroom environment. The design of the program is based on the ADDIE model (Analyze, Design, Develop, Implement, Evaluate), which is frequently used in curriculum development literature and is known for its systematic and cyclical structure. The ADDIE model is a powerful tool, especially in multilayered and conceptual development-focused areas, as it addresses the instructional design process in stages and allows for data-driven decision-making at each stage (Branch, 2009; Molenda, 2003; Shrivastava et al., 2019). The program is structured according to the five fundamental stages of the ADDIE model:

- Analysis – Determining the current status and areas for development of the teachers.
- Design – Planning the program's objectives, content, and activities.
- Development – Creating the activities, materials, and assessment tools.
- Implementation – Carrying out the program with teachers in a classroom setting.
- Evaluation – Determining the program's effectiveness and making improvements based on feedback.

The program proposed in this article aims to contribute to addressing the conceptual deficiencies observed in algebra instruction in Turkey, enrich teachers' algebraic thinking pedagogies, and ultimately strengthen students' mathematical understanding.

Conceptual Framework

Algebraic Thinking

Algebraic thinking is a fundamental component of mathematics, playing a critical role not only in teaching algebra topics but also in mathematical reasoning, problem-solving, and generalization processes. Kieran (2007) defines algebraic thinking across three main dimensions: (i) generalizing, (ii) relating and modeling, and (iii) symbolic representation and manipulation. This definition emphasizes that algebraic thinking is not limited to symbolic operations but also involves students understanding mathematical structures within a broader context. Kaput (2008), on the other hand, views algebraic thinking as the processes through which students connect mathematical expressions and explain these expressions using different representations. For instance, noticing a regularity in an arithmetic problem and generalizing it develops students' algebraic thinking skills. In this context, it is important to encourage students to generalize and discover mathematical relationships from an early age.

Research conducted in the Turkish context (Akkan & Çakıroğlu) shows that the algebraic thinking levels of pre-service teachers are open to development. Similarly, Kieran (2004) emphasizes that algebraic thinking has a multidimensional structure and therefore needs to be systematically supported during the instruction process. Candidates often stick to prototypical solutions, remain superficial in generalization processes, and use symbolic representations independently of conceptual meaning. This situation reveals the fact that improving teachers' own algebraic thinking skills will directly reflect on students' learning processes.

Algebraic Habits of Mind (AHoM)

An important theoretical framework supporting algebraic thinking is the Algebraic Habits of Mind (AHoM) approach. Introduced by Driscoll (1999), this approach defines the guiding questions and strategies teachers can use to direct students' mathematical thinking. AHoM consists of three main dimensions and various components:

- Doing and Undoing: This dimension refers to students' ability to think forwards and backwards in mathematical processes. For example, being able to think about the inverse of an operation while performing it deepens students' conceptual understanding.
- Building Rules to Represent Functions: Students exploring functional relationships by transitioning between different representations (table, graph, algebraic expression) forms the basis of this dimension. This process allows students to see relationships between mathematical structures and make generalizations.

- Abstracting from Computation: Students are expected to develop general mathematical principles based on operations, rather than focusing solely on operational solutions. This enables students to generalize independently of calculations.

These three dimensions allow algebra instruction to move beyond a focus on symbol manipulation and enhance students' mathematical reasoning and problem-solving skills.

AHoM in the Context of Teacher Education

Teachers play a critical role in the development of algebraic thinking. The guiding questions teachers use direct students' thinking processes and support their construction of conceptual understanding. The literat) has shown that activities prepared by teachers considering the AHoM dimensions positively influence students' mathematical reasoning. For example, the question "Can you reverse this operation?" supports the doing/undoing dimension, while "Can you generalize the pattern in this table?" encourages building functional relationships. Therefore, it is clear that teachers need professional development support in developing and implementing such questions.

The Importance of Professional Development Programs

The professional development of mathematics teachers should aim not only to increase content knowledge but also to strengthen pedagogical content knowledge (Shulman, 1986). In this context, AHoM-based programs directly contribute to teachers' classroom practices. By recognizing AHoM, teachers can develop strategies to support algebraic thinking in students. In Turkey, there are criticisms that in-service professional development programs often remain theoretical and do not offer application opportunities to teachers (Baydar, 2021). Therefore, the unique aspect of the proposed program is that it offers an activity-based, interactive, and reflective structure.

Program Design: AHoM-Based Professional Development Program

The professional development program proposed in this study aims for pre-service and in-service teachers to deeply understand algebraic thinking processes and develop strategies to support these processes in their classroom practices. The program is designed based on the Algebraic Habits of Mind (AHoM) framework and consists of a total of eight sessions. Each session is structured in an integrated manner with content, instructional activities, and assessment processes in line with determined learning outcomes.

Findings and Discussion

The examples used throughout the trainings were adapted from Driscoll's book "Fostering Algebraic Thinking."

1st Week: Algebra Conceptually and its Historical Development

The first session of the program is structured to reveal participants' current level of knowledge regarding the concept of algebra and to contribute to their understanding of algebra's historical development. This session emphasizes that algebra is not a narrow field reduced solely to operating on symbols, but rather a rich thinking domain involving higher-level mental processes such as abstraction, generalization, relating, and modeling. The aim is for participants to recognize that algebra has been defined and developed in various forms across different geographies and periods in history. Thus, it is intended that they better interpret the place and weight of algebra in current mathematics curricula.

Learning Outcomes:

- Be able to explain the definition and distinguishing features of algebra.
- Be able to make sense of the algebra-arithmetic relationship.
- Be able to interpret the different definitions and developmental lines of algebra throughout the historical process.

- Be able to evaluate the place and importance of algebra in current curricula.

Learning-Teaching Process: The session begins with an informative presentation. This presentation touches upon the meaning of the algebra concept in different contexts, its historical origins, and its interaction with other sub-fields of mathematics. Participants are then guided to discuss example situations regarding the relationship between arithmetic and algebra. During the discussion process, participants are encouraged not only to recall correct definitions but also to make interpretations based on their own learning and teaching experiences. This ensures that participants deeply understand the concept and reconstruct it pedagogically.

Assessment: At the beginning of the session, open-ended diagnostic questions are posed to reveal participants' readiness for the topic. Through these questions, the scope and quality of the prior knowledge participants possess about the concept of algebra are determined. The in-class discussions conducted throughout the session are monitored under the scope of formative assessment, and how teachers structure their thoughts and interpret each other's contributions are analyzed. Thus, the assessment is carried out based on an approach that centers the process, not just the product.

In the first session, teachers addressed the conceptual foundations and historical development of algebra. This foundational perspective prepares the necessary ground for them to understand the theoretical frameworks of algebraic thinking and the AHoM dimensions in the second session.

2nd Week: Algebraic Thinking and Related Theoretical Frameworks

The second session of the program aims for participants to approach the concept of algebraic thinking from a multidimensional perspective and to grasp how this type of thinking can be supported through classroom practices. Accordingly, both the developmental stages of algebraic thinking and the Algebraic Habits of Mind (AHoM) framework are introduced in detail. The session prepares the ground not only for teachers to acquire conceptual knowledge but also to develop pedagogical strategies that will guide students' thinking processes. Learning Outcomes:

- Be able to explain the developmental stages of algebraic thinking.
- Be able to recognize the basic dimensions and components of AHoM.
- Be able to develop guiding questions based on AHoM and adapt them to classroom practices.

Content: Within the scope of the session, the three main dimensions of AHoM are discussed in detail:

- Doing and Undoing: Being able to think forwards and backwards in mathematical operations.
- Building Rules to Represent Functions: Being able to form rules by transitioning between different representations (e.g., table, graph, algebraic expression).
- Abstracting from Computation: Being able to derive general principles from operational processes.

Concrete classroom examples are provided for each dimension, and teachers are enabled to discuss these examples. Furthermore, participants are provided with example guiding questions that support these dimensions; collective evaluations are made regarding the impact of these questions on students' thinking processes.

Activities: Participants are asked to create in-class questions that can support the AHoM dimensions, based on their own teaching experiences or observations. These questions are shared and discussed within group work. During the discussions, participants analyze which AHoM dimension the questions support, what kind of thinking process they might trigger in students, and their potential strengths/weaknesses.

Assessment: At the end of the session, the questions developed by the teachers are examined in terms of how much they align with the AHoM dimensions. This process is conducted with a formative assessment approach; the goal is less to measure the quality of the questions produced by the teachers and more to make their ways of thinking visible and to develop them. Analyzing the questions developed by participants at the group level increases professional sharing and contributes to teachers learning from each other.

The second session focused on the dimensions of algebraic thinking and the components of AHoM. From this point onward, in the third session, teachers will experience, through concrete activities, how they can support students' transition from operational thinking to algebraic thinking.

3rd Week: Transition from Algorithmic Thought to Algebra

The third session of the program aims for teachers to grasp how they can facilitate students' transition processes from operational and algorithmic thinking to algebraic thinking. Students often solve problems merely by following specific algorithms; this limits their development of skills central to the essence of algebra, such as generalization, symbolization, and inverse operation thinking. In this session, the goal is for teachers to develop strategies that support the shift from algorithmic-based solutions to more flexible, conceptual, and algebraic approaches.

Learning Outcomes:

- Be able to distinguish between algorithmic and algebraic thinking.
- Be able to develop smooth transition strategies to facilitate the shift from algorithmic to algebraic thinking.
- Be able to analyze student solutions and evaluate them in the context of algebraic thinking.

Activities: The "Golden Apple" and "Postage Stamp" activities are specifically implemented in this session. If we evaluate the Golden Apple activity as an example, it stands as a problem that supports inverse operation thinking, equation setting, and generalization skills. Now, let's examine this example closely:

A man gathers a basket full of golden apples from a magical garden. Three guards he meets on the way, in turn, each ask for half of the remaining apples in his basket plus two more apples. The man continues his journey after giving what was requested each time. After the last guard, only 2 apples remain in his basket.

According to this: a) How many apples did the man gather initially? b) If 4 apples had remained, what would the initial amount have been? c) If 6 apples had remained, what would the initial amount have been?

This problem requires students not only to perform forward algorithmic calculations but also to use inverse operations to find the initial value. This process particularly supports the "doing/undoing" dimension of AHoM. Students are guided to go beyond step-by-step execution and develop equation-setting and generalization strategies.

During the activity, teachers consider different student solutions, generate alternative solution paths, and relate these paths to the AHoM dimensions. Group discussions provide an opportunity for teachers to analyze not just the correct answer, but also the logic of the process, the strategies used, and potential errors.

Assessment:

The assessment process does not focus solely on the correctness of the final result. Instead, consideration is given to how teachers structure their problem-solving processes, which strategies they use, and how they interpret student errors. The solution strategies developed by teachers in group discussions are analyzed in terms of how closely they align with the AHoM dimensions.

The third session focused on the differences between operational and algebraic thinking. Following this transition, the fourth session will focus on how number sense and number theory can be used in the development of algebraic thinking.

4th Week: Revealing Algebraic Thinking Using Elementary Number Theory

In this session, number sense and number theory are linked to the development of algebraic thinking; the goal is for teachers to develop strategies that support the transition from number sense to algebra. Number sense allows students to perceive numbers not just as objects of operation, but also in the context of structures and relationships. Problem situations carried out through the factors of numbers offer important opportunities for this transition.

Learning Outcomes:

- Be able to explain the relationship between number sense and algebra.
- Be able to develop instructional designs that facilitate the transition from number sense to algebra.

Activities: The "Nu Function" and "Sheep Eric" activities are implemented in this session. Let's examine the Nu Function activity:

The Nu Function is a special function that counts the factors (divisors) of a number. For example, the factors of the number 6 are 1, 2, 3, and 6; therefore $\nu(6)=4$. Participants are directed the following questions in the activity:

$$\nu(5)=?$$
$$\nu(12)=?$$
$$\nu(24)=?$$
$$\nu(23\cdot32\cdot54)=?$$

Find several inputs for the function that have an output of 6. Classify all numbers n for which $\nu(n)=3$.

This activity aims to explore functional relationships through the factors of numbers and to structure the concept of a function. Students do not only calculate $\nu(n)$ for specific n values, but also engage in inverse thinking processes, such as "finding the input of a function given its output." Thus, the AHoM dimensions of doing/undoing and building rules to represent functions are supported. Furthermore, classification questions like $\nu(n)=3$ allow students to encounter the concept of infinite sets and make generalizations.

Assessment: At the end of the session, teachers are asked to create their own activity designs that will support the transition from number sense to algebra. Participants present these designs in a group setting and make improvements based on peer feedback.

The fourth session examined the connections between number sense and algebra. This context opens the door for the fifth session to activities that will support students' ability to generalize through functional relationships and justify these generalizations with convincing explanations.

5th Week: Discovering and Justifying Generalizations about Functional Relationships

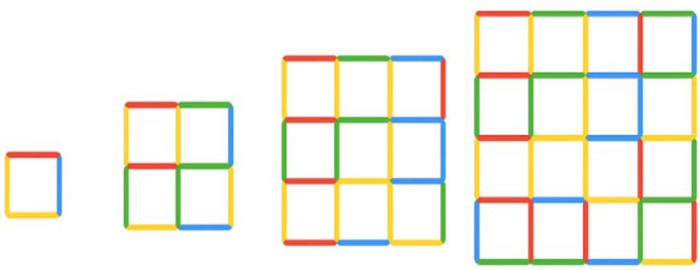
The fifth session addresses the skill of generalization, which is central to algebraic thinking, and how this skill can be integrated with justification processes. In mathematics, generalization allows students not only to operate on specific examples but also to recognize broader rules and structures based on these examples. Furthermore, students justifying the generalizations they develop through convincing explanations is a critical step that strengthens the conceptual depth of algebraic thinking.

Learning Outcomes:

- Be able to make generalizations about functional relationships.
- Be able to develop convincing explanations in the process of justifying generalizations.
- Be able to recognize the difficulties students may encounter while generalizing.

Activities: Two activities stand out in this session: "The Locker Problem" and "Toothpick Squares." Both activities encourage students to recognize patterns, establish functional relationships, and generalize them. The Toothpick Squares Activity is a powerful tool that enables students to generalize from visual models. The activity is presented as follows:

Below are some steps of "growing" squares made from toothpicks.



1. Draw the figure for the fifth step.
2. How many small squares does the figure in the fifth step consist of?

3. How many small squares does a large square with 10 toothpicks on each side consist of?
4. Write a rule to find the number of small squares in any large square.
5. How many small squares are there in the large square obtained using 112 toothpicks?

This problem requires students not only to perform operations but also to discover patterns and develop formulas by generalizing these patterns. For example, some students may use a table approach to examine numerical increases, while others may use visual representations to express the total number of toothpicks with a formula. In this process, students have the opportunity to express generalizations in different ways:

- Table Approach: Students table the number of toothpicks for a shape of length n squares, examine the increases between the columns, and derive a general rule from there.
- Non-Table (Visual) Approach: Students draw the squares, separate the internal and external toothpicks, and develop a rule that holds for the general case based on this separation.

Both approaches activate students' algebraic thinking in different dimensions. The table approach leads to generalization through repeated change in patterns, while the visual approach offers a more structural analysis. Teachers, by making these different approaches visible in the classroom, help students relate the generalizations they produce through different pathways. The activity also highlights the skill of justification. Students do not just write rules; they must explain why these rules are valid in every case. At this point, teachers support the process with questions such as, "How can you show why this rule works?" or "Does this explanation convince you?"

Assessment: In this session, the explanations developed by the teachers are analyzed through open-ended questions. The assessment focuses not only on whether the correct formula is found, but also on the persuasiveness of the explanation and the thinking process on which the generalization is based. Participants evaluate each other's explanations, discussing strengths and weaknesses. In this way, teachers both experience the generalization process and have the opportunity to pre-emptively notice the typical difficulties students may experience in this process.

The fifth session discussed functional relationships and justification processes. Building on this, the next session will deepen the relationship between operations and structures in algebra, and teachers will experience how to transition from operations to structural generalizations.

6th Week: Discovering Generalizations about Operations and Structures

This session emphasizes that algebra is not merely composed of symbolic expressions, but is a domain involving generalization processes related to operations and structures. The ability of students to make generalizations based on different operational examples and to connect these generalizations with mathematical structures is of critical importance for the development of algebraic thinking. The objective of this session is for teachers to experience such generalization processes, recognize the typical difficulties students face, and develop pedagogical strategies for this process.

Learning Outcomes:

- Be able to develop generalization approaches regarding operations and structures.
- Be able to analyze the difficulties students may face in the generalization process.

Activities: Two activities stand out in this session: "Finding Values in a Table" and "Windows of the Skyland Building."

The Windows of the Skyland Building Activity is a powerful problem situation that supports students' transition from operational thinking to generalization:

In a 12-story building at Skyland Istanbul, there are 38 windows on each floor. The cleaning cost for the windows starts at 2 TL per window on the first floor and increases by 0.5 TL on each subsequent floor. Can all windows be cleaned with a total budget of 2500 TL? If the budget is insufficient, how much is the shortfall? If the building were 30 stories high and a budget of 40,000 TL was allocated, would this budget be sufficient?

This problem allows students to initially attempt an arithmetic approach. For example, the number of windows on each floor can be multiplied by the cost per floor, and the results can be added up individually. However, this method becomes cumbersome and inefficient in more complex scenarios, such as the 30-story case. At this point, teachers guide students to find more general and shorter ways, i.e., to develop algebraic generalizations. The following types of guiding questions can be directed to students:

- "Can we create a table to see the change in this group of numbers?"
- "Is there an easier way to add the numbers?"
- "What kind of shortcut can you use for larger numbers?"

For instance, for the sequence of increasing values: 2.0 – 2.5 – 3.0 – 3.5 – 4.0…, students can develop a shorter method by finding the average of these numbers and multiplying it by the number of floors. This approach paves the way for noticing the sum of arithmetic sequences and reaching a more general algebraic expression. The pedagogical power of the activity lies in allowing students to initially approach the question with operational thinking, but then encouraging them to generalize and abstract. This specifically supports the AHoM dimensions of abstracting from computation and building rules to represent functions.

Assessment: The solution strategies developed by the teachers in this activity are evaluated through group discussions. Participants focus not only on the result but also on the mathematical logic of the paths followed to reach the result and their generalization potential. In this way, teachers gain awareness for their own classroom practices by experiencing in advance the processes of students developing shortcuts, noticing patterns, and formulating these patterns.

The sixth session addressed generalization processes through operations and structures. Building on this foundation, the seventh session will focus on developing the sense of symbol, which is central to algebraic thinking, and analyzing the difficulties students encounter in the symbolization process.

7th Week: Revealing Symbol Sense in Algebraic Thinking

The seventh session addresses the role of symbols in algebraic thinking and the difficulties students face in transitioning to the use of symbols. Symbols are not merely short notations for mathematical expressions; they are also a critical tool for generalization, representing relationships, and facilitating transitions between different representations. Recognizing the difficulties students experience in the symbolization process and developing strategies to facilitate this process form a core dimension of the teacher's guidance role.

Learning Outcomes:

- Be able to grasp the importance of symbol use in algebraic thinking.
- Be able to analyze the difficulties students face in the symbolization process.
- Be able to develop strategies that facilitate the transition to symbol use.

Activities: The session addresses how symbol use can be structured through the "Consecutive Numbers" and "Finding the Price" activities.

The Sum of Consecutive Numbers Activity is conducted through the following questions:

$$1+2+3+4=10$$
$$2+3+4+5=14$$
$$3+4+5+6=18$$
$$4+5+6+7=22$$

1. Find the relationship between the sum of four consecutive numbers and the numbers being added by trying different numbers.
2. Calculate the result of the operation 499+500+501+502 using a shortcut.
3. Develop a rule that gives the sum of four consecutive numbers in the general case and express it algebraically.
4. Can the number 296 be written as the sum of three consecutive numbers? Why?
5. Repeat the same steps for two, three, and five consecutive numbers.
6. Find shortcuts to write the numbers 45, 57, 62, 75, and 80 as the sum of two or more consecutive numbers.

This activity encourages students to generalize from concrete numerical examples, and then to convert these generalizations into symbolic representations. While participants initially arrive at the result with arithmetic operations, with teacher guidance, they learn to develop shorter, general, and effective solutions using symbols.
Interpretation of the Activity with AHoM Dimensions: The consecutive numbers sum activity specifically supports two important dimensions:

- Developing the habit of using symbols – Students learn to abstract by using expressions like "a,a+1,a+2,…" to represent consecutive numbers.
- Working with equivalent symbolic expressions – Students compare different symbolic representations, choose the most appropriate one, and try to justify their generalizations.

For example, in Doris's work, the sum of four consecutive numbers was expressed as "2[a+(a+3)]," and this expression was then simplified to "4a+6." This experience gives students the opportunity not only to develop a formula but also to make choices between equivalent symbolic expressions and to justify these choices. Thus, it is revealed that the use of symbols is not just a tool that shortens operations, but also a valuable component of mathematical thinking.

Assessment: The strategies developed by the teachers are discussed in terms of their usability in classroom practices. In this process, the focus is not only on finding the correct formula but also on how meaningful and convincing the symbolic expressions are for the students.

The importance of symbol use in algebraic thinking was discussed in the seventh session. In the final session of the program, the focus will be on teachers deepening students' conceptual understanding by establishing connections between multiple representations; thus, all dimensions addressed in previous sessions will be completed with a holistic perspective.

8th Week: Establishing Connections Between Multiple Representations in the Development of Algebraic Thinking

The final session of the program addresses the critical role of multiple representations in the development of algebraic thinking. The ability to represent algebraic concepts not only with symbolic expressions but also with tables, graphs, models, verbal descriptions, and dynamic visuals contributes to students' deeper understanding of the concepts. Transitions between representations are crucial, especially in grasping the functional nature of algebra and establishing connections between different mathematical fields (arithmetic, algebra, geometry).

Learning Outcomes:

- Be able to grasp the importance of multiple representations in algebraic thinking.
- Be able to recognize the difficulties students face in transitioning between representations.
- Be able to use technology-supported environments to create multiple representations and transition between them.

Activities: The "Rounding Cape Horn" (or Circumnavigating the Horn) activity is implemented within the scope of the session. The activity is presented below:

Assume that the ships carrying families migrating from New York to San Francisco in the 1800s departed on the first day of every month, and the voyage lasted 6 months. Imagine you are on a ship departing from New York. How many ships arriving from San Francisco will you encounter during this voyage? This problem requires students to think not only with verbal expressions but also through timelines, diagrams, or table representations. For example, the solution process can be facilitated by tabulating the movements of the ships by month or by showing the positions of the ships moving in one direction with visual diagrams. Students arrive at the same result using different representations, which reveals how transitions between representations enrich thinking.

The activity specifically activates the AHoM dimensions of building rules to represent functions and abstracting from computation. While students initially think through the concrete scenario step-by-step, they later move on to generalization by considering questions such as:

- Generalization: "How many ships would be encountered on a voyage lasting n months?"

- Variable Speed Scenario: "How many ships would be encountered if ships in one direction were twice as fast as the others?"

Such extensions enable students not only to solve the problem situation in a specific context but also to reach more general and abstract rules. It is possible to visualize different representations in the activity using GeoGebra, dynamic table programs, or graphical simulations. Thus, students have the opportunity to explore the same mathematical relationship with different tools.

Assessment: Teachers' awareness of multiple representations is measured through open-ended questions and reflective evaluation forms. Participants are expected to justify the solutions they produce using different forms of representation and discuss which representations might be more explanatory for students.

Conclusion and Implications

This study demonstrates the potential of the methodological approach centered on the Algebraic Habits of Mind (AHoM), offered by the eight-session professional development (PD) program, to deepen not only teachers' conceptual knowledge but also their Pedagogical Content Knowledge (PCK) (Shulman, 1986), which is the cornerstone of effective teaching. The AHoM framework, which forms the core of the program, has undertaken the mission of making mental processes visible and developing targeted instructional scaffolding strategies, as proposed by Driscoll (1999) and Driscoll et al. (2001). The activity-based approach in every session has allowed teachers to both experience potential student thinking pathways and systematically analyze these thoughts within the dimensions of AHoM. This approach addresses criticisms often directed at traditional, purely theoretical PD models (Baydar, 2021), bridging the critical gap between abstract theoretical knowledge and concrete classroom application.

The methodological strength of the program stems from the integration of experiential learning and reflective practiceprinciples. By engaging in a cycle of activity implementation, peer discussion, and strategy analysis, teachers transition into a continuous learning community, which fosters the sustainability of professional growth (Stephens et al., 2012). Furthermore, the systematic structuring of the program based on the ADDIE model (Branch, 2009; Molenda, 2003) ensures that the development process is traceable, rigorous, and adaptable, offering a replicable model for other content areas and making a significant contribution to the literature on PD curriculum design.

The pedagogical advancements of this program provide a sensitive intervention against the known challenges in algebra instruction. Activities in Sessions 3 and 4 directly address students transition from operational/algorithmic thinking to abstract algebraic reasoning (Kieran, 2007). By utilizing contexts like the "Golden Apple" and the "Nu Function," the Doing/Undoing and Abstracting from Computation dimensions of AHoM are strengthened. Similarly, tasks in Sessions 5 and 6 ("Toothpick Squares," "Skyland Windows") compel teachers to move beyond finding *a* rule to justifying *why* the rule holds true structurally. This aligns with Kaput's (2008) view that algebraic thinking is fundamentally about establishing and representing relationships and structures. Finally, Sessions 7 and 8 focus on developing symbol senseand representational fluency, equipping teachers with strategies to make symbolic expressions meaningful and conceptually grounded. Given that difficulties in symbolization and translation across multiple representations (symbolic, graphical, tabular, verbal) are common barriers for students (Kieran and Yerushalmy, 2004), the program enhances teacher capacity in overcoming these obstacles.

In summary, this AHoM-centered program offers a robust, conceptually driven, and practice-oriented framework, responding to documented deficiencies in algebraic teaching practices in Turkey and beyond (Akkan & Çakıroğlu, 2012). By significantly increasing teachers' capacity to guide students toward deeper mathematical understanding, this model is expected to create a positive and sustainable long-term impact on student learning outcomes and the overall quality of mathematics instruction.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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