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Utilization of Algebraic Habits of Mind According to Types of Algebraic Demands

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Abstract: This study aims to explore the utilization of algebraic habits of mind according to types of algebraic demands. To this end, algebraic demands in middle school mathematics textbooks were examined and evaluated through content analysis, employing a document analysis design. The research design of this study is document analysis. The findings indicate that algebraic demands can be categorized into four groups: (1) demands requiring the construction of rules, (2) demands requiring the application of a constructed rule to a specific case, (3) demands requiring the validation of a rule, and (4) demands requiring the use of a known rule. Results show that different types of algebraic demands directly influence the ways in which algebraic habits of mind are activated. For instance, when students are asked to explain why a given rule holds, explicit justification is expected; however, when students are engaged in constructing the rule themselves, justification emerges naturally within the process. Thus, the same habit of mind may function differently depending on the nature of the algebraic demand. In conclusion, the study highlights that algebraic thinking extends beyond the mechanical application of rules. It emphasizes the importance of exposing students to diverse types of algebraic demands in order to enrich their algebraic habits of mind. Accordingly, teachers are encouraged to develop awareness of these habits, to critically consider the nature of algebraic demands in textbooks, and to design learning environments that foster their effective development.

Keywords: Algebraic thinking, Algebraic habits of mind, Type of algebraic demands.

Introduction

Algebra is a fundamental domain of mathematics that involves analyzing functional relationships, examining representational systems derived from these relationships, and addressing number systems, unknowns, patterns, as well as the formulation and solution of equations. Algebraic thinking, in turn, can be defined as the ability to generalize algebraic operations, relationships, and patterns; to make inferences from such generalizations; and to express them through appropriate representations. A review of the literature reveals that studies have been conducted on scaling up students' algebraic thinking skills (Chimanoi et al., 2018; Kaput, 1999), developing them (Driscoll, 1999), and determining their levels of algebraic thinking (Hart et al., 1998). Based on Cuoco, Goldenberg, and Mark's (1996) study of useful ways of thinking about mathematical content, which they defined as habits of mind, Driscoll and Moyer (2001) introduced the framework of algebraic habits of mind to conceptualize the processes underlying algebraic thinking. Within this framework, the key habits are classified as Building Rules to Represent Functions, abstracting from computation, and doing–undoing. Each of these habits comprises several sub-habits and plays a crucial role in fostering algebraic thinking.

Algebraic Habits of Mind

According to Driscoll (1999), possessing algebraic thinking requires thinking about functions and how they work, as well as considering the effect of a system's structure on calculations. These two aspects of algebraic thinking are facilitated by certain habits of mind. In this framework, Driscoll (1999) classifies the habits of mind in algebra into three categories: doing–undoing, building rules to represent functions, and abstracting from computation. Each habit contains components that promote algebraic thinking and the meaningful interpretation of algebraic problems (Figure 1).

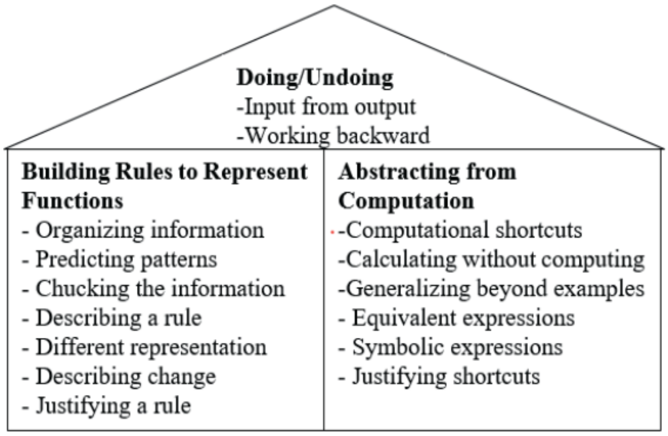


Figure 1. Algebraic habits of mind (Driscoll & Moyer, 2001)

Doing and undoing refers to the symbolic manipulation used to solve, write, or reconstruct mathematical expressions. Students should not only be able to carry out an algebraic operation to find its result but also work backward from the result to reach the starting point. For example, just as they can solve the equation $3x^2 - 12 = 0$, they should also be able to construct an equation whose roots are $x = 2$ and $x = -2$. Through this habit of mind, students focus not only on obtaining the final answer but also on reflecting upon the process itself.

The building rules to represent functional habits are largely related to students' ability to search for and define functional relationships. This habit consists of seven components, as illustrated in Figure 1. Students who possess these components use their habits of mind to identify relationships within algebraic problems. According to Driscoll and Moyer (2001), when an individual encounters a problem, he/she uses the habit of "The building rules to represent functions" by trying to understand and developing strategies for solution, determining relationships between quantities, using representations, performing operations between representations, searching for patterns, finding the rule of the pattern, and defining the general rule using representations.

The abstracting from computation habit refers to the ability to think about computations independently of the specific calculations being performed. This habit consists of six components, as shown in Figure 1. Learners who exhibit this habit have an understanding of algebraic structures that allows them to develop shortcuts and generalizations about computations. Abstraction is a key element of this habit of mind. It involves the process of extracting mathematical objects and relationships based on generalization (Lew, 2004). For example, when calculating the sum $1 + 2 + 3 + \ldots + 40$, students may regroup the numbers to make 41, as in $40 + 1 = 41$; $39 + 2 = 41$; $38 + 3 = 41$, and so on, ultimately reaching the result through reasoning. It is important to allow students to think in different ways and find alternative solutions during this process.

The purpose of the study is to examine and classify algebra-related questions and activities found in mathematics textbooks and supplementary resources in terms of algebraic demands. In addition, the study aims to analyze the ways in which habits of mind in algebra are exhibited according to these types of demands.

Method

This study was conducted using the document analysis method, which is one of the qualitative research approaches. For the purpose the study, the cognitive and structural characteristics of algebraic tasks in the selected resources were identified and then categorized according to a defined analytical framework.

The data set of the research consists of 4 middle school mathematics textbooks and supplementary materials. In selecting the sources, the criterion of covering algebra topics was taken into consideration. In each book, the sections containing algebra topics were examined, and the examples, exercises, activities, and assessment questions included in these sections were determined as the units of analysis.

The data were analyzed using the content analysis method. The analysis process was carried out based on four categories of algebraic demands described below. To ensure the validity of the research, the analysis process was documented in detail. Based on the feedback received from the experts, the coding criteria were revised, and the final analysis was conducted accordingly.

Results and Discussion

Types of Algebraic Demands

An analysis of the algebraic demands found in sources containing algebraic content revealed that these demands can be grouped under four main categories:

- *Demands requiring the discovery of a rule:* Tasks that require students to examine relationships among given situations and discover a general rule or pattern.
- *Demands requiring the discovery and application of a rule to a specific case: Tasks* that require students to first determine a general relationship and then apply it to a particular example or problem.
- *Demands requiring the verification of a rule:* Tasks in which students are asked to demonstrate, prove, or test the validity of a given algebraic rule or generalization by using counterexamples.
- *Demands requiring the use of a known rule:* Tasks that require students to directly apply a previously learned algebraic rule, algorithm, or operation.

OSCA (Organize, Search for Relationships, Construct, Apply) Model

The OSCA framework involves a sequence of processes in which students first search for patterns or relationships within given data, then intuitively predict a generalization, construct this generalization through an algebraic expression or structure, and finally validate their proposition with mathematical reasoning. This framework was developed by directly adapting the dimensions of Driscoll's ZCA framework to the context of classroom instruction. In this way, Driscoll's theoretical framework has been transformed into a functional dimension within teachers' guidance strategies. Each step of the OSCA model has been directly integrated into the module content and activity structure of the instructional program.

The OSCA framework was used as an analytical tool aimed at explaining how each habit of mind manifests in students' thinking and at identifying how teachers can recognize and support these processes. For instance, within the habit of *Building Rules to Represent Functions*, a pedagogical structure was developed to help teachers evaluate processes such as how students identify patterns, establish relationships through different representations, and derive generalizations from these relationships.

Table 1. Distribution of the components of the "Building Rules to Represent Functions" habit according to the OSCA model

Step	Related AHoM components
Organize	Organizing information, Chucking the information, Different representations
Search for Relationships	Describing change, predicting pateens/rules
Construct	Describing a rule (algebraic expression, equation, inequality, etc.), justifying the rule
Apply	Applying to a special case

Similarly, the pedagogical structure developed for the *"Abstracting from Computation"* habit aimed to make visible how students abstract algebraic structures independently of specific operations and how they reconstruct and use these structures in problem solving.

Table 2. Distribution of the components of the “Abstracting from Computation” habit according to the OSCA model

Step	Related AHoM components
Organize	Organizing information, Computational shortcuts, calculating without computing, Chucking the information,equivalent expressions
Search for Relationships	Describing change, predicting pateens/rules
Construct	Generalizing beyond examples/defining rules, (algebraic expression, equation, inequality, etc.), justifying the resulting rules/generalizations,
Apply	Applying to a special case

This structure not only provided a theoretical classification but also directly guided the design of the program’s activities. For example, in the session titled *“Exploring and Justifying Generalizations about Functional Relationships,”* participants were expected to describe changes within patterns, make predictions, and justify their generalizations; these goals were supported through activities structured according to the corresponding stages and actions. Similarly, in the session themed *“Sense of Symbol,”* abstraction processes such as recognizing equivalent expressions and transforming between these expressions were explicitly established as instructional objectives. Through this holistic structure, the developed program served not merely as a means of content delivery but as a *strategic roadmap* for how teachers interact with their students. Throughout the entire process, the project team collaboratively managed the design through face-to-face meetings, iteratively revising the module designs and activity structures. Each content component was refined through the contributions of at least two different team members, and decisions were made collectively rather than individually. In particular, issues such as diversifying instructional strategies, adapting activities to teachers’ classroom realities, and concretizing the AhoM (Algebraic Habits of Mind) components in instructional contexts were shaped through numerous collaborative revision processes.

The Relationship Between Algebraic Demand Types and the OSCA Model

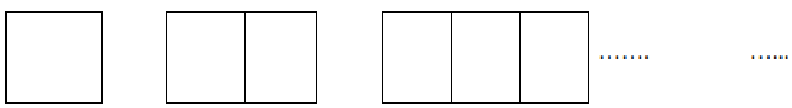
As illustrated in Table 3, the sequence of steps within the OSCA framework may vary depending on the type of algebraic demand. When a rule is directly applied, learners may be required to justify *why* the rule is valid. However, when the learning task involves constructing the rule through exploration, the justification process becomes inherent to the activity itself, and therefore a separate justification step may not be necessary. This variation indicates that the structure of the OSCA model is not static but dynamically adapts to the cognitive and procedural requirements of each algebraic demand type. In other words, the *primary algebraic demand* determines the reasoning pathway that students are expected to follow. Accordingly, teachers can use the OSCA framework to anticipate which cognitive actions—comprehending, searching for relationships, constructing, or applying—are most relevant for guiding students’ algebraic thinking within a given task.

Table 3. Relationship between Algebraic demands of tasks types and OSCA steps

	Organize	Search for Relationships	Construct	Apply
Demands requiring the construction of rules	+	+	+	
Demands requiring the application of a constructed rule to a specific case	+	+	+	+
Demands requiring the validation of a rule	+	+	+	
Demands requiring the use of a known rule	+			+

According to the *demand type that requires discovering a rule*, the stages of addressing this demand can be illustrated through examples related to both the *habit of building rules to represent functions* and the *habit of abstracting from computation*.

Example (Habit of Building Rules to Represent Functions):



The squares in the pattern above are created with matchsticks. How many matchsticks are used in any given step?

Solution	AHoM compenents	OSCA compenents
A pattern is given in which 3 more matches are used in each step than in the previous step. It asks us to find how many matchsticks were used in the nth step.	Organizing information	
Let's illustrate it with a table. Step: 1. 2. 3. 4. 5. 6. 7. Number of matchsticks: 4 7 10 13 16 19 22	Different representations	Comprehend
Let's use the equivalent expression to see the relationship between the number of steps and the matchstick. step: 1. 2. 3. 4. … N Number of matchsticks: 4 7 10 13 … Relationsships: 4 4+3 4+6 4+9 … 4+3.0 4+3.1 4+3.2 4+3.3 4+3(n-1) In each step, 3 more matches were used than the number of matches used in the previous step.	Describing change, predicting pateens/rules	Search for Relationships
From the table, continue by adding three times of each step number and 1. Algebraic expression: 4+3(n-1)=3n+1	Organizing information	Construct

Example (Habit of abstracting from calculation): 1 + 3 + 5 + 7 + 9 + 11 … Find the rule that gives the sum of consecutive odd numbers.

Solution	AHoM compenents	OSCA compenents
We are asked to find the rule that gives the sum of odd numbers.	Organizing information	
İFirst number / The sum of the first 2 numbers / The sum of the first 3 numbers / The um of he first 4 numbers / The su of the first 5 numbers / … 1 / 4 / 9 / 16 / 25 / …	Different representations	Comprehend
İFirst number / The sum of the first 2 numbers / The sum of the first 3 nu bers / The sum of the first 4 numbers / The sum of the first 5 numbers / … / N 1 / 4 / 9 / 6 / 25 / … / … 1^2 / 2^2 / 3^2 / 4^2 / 5^2 / … / n^2	Describing change	Search for Relationships
The result of the sum is the square of the step number at each step. Accordingly, the sum of n consecutive odd numbers starting from 1 is n^2.	Describing a rule	Construct

Conclusion

The findings of this study reveal that algebraic demands in instructional materials can be effectively analyzed through the combined use of Driscoll's (1999) Habits of Mind in Algebra framework and the OSCA (Comprehend, Search for Relationships, Construct, Apply) model. This integration provides a comprehensive perspective on how algebraic thinking emerges and can be supported within classroom practice.

Driscoll's framework emphasizes that students' algebraic reasoning develops through specific cognitive habits—namely doing–undoing, building rules to represent functions, and abstracting from computation. The present study extends these theoretical dimensions by demonstrating how each habit can be operationalized within the OSCA model, which structures students' algebraic engagement as a cyclical process of understanding, relational reasoning, construction, and application. In this respect, the OSCA framework not only translates Driscoll's theoretical ideas into classroom practice but also serves as a pedagogical roadmap for teachers seeking to foster deeper algebraic thinking.

The analysis of different algebraic demand types—such as discovering rules, applying rules to specific cases, verifying rules, and using known rules—suggests that each demand requires a distinct combination of cognitive and metacognitive actions. For instance, when students are engaged in tasks that require discovering a rule, they tend to activate both the building rules to represent functions and abstracting from computation habits. Conversely, tasks requiring the application or verification of known rules primarily mobilize procedural reasoning and justification. This alignment indicates that the type of algebraic demand determines the pathway of reasoning within the OSCA sequence, echoing findings by Kaput (2008) and Kieran (2007), who similarly argued that the nature of algebraic activity shapes students' cognitive engagement.

Moreover, the iterative design process of the instructional modules demonstrated that embedding habits of mind into task structures enables teachers to move beyond procedural instruction toward reflective practice. The collaborative revisions conducted by the project team ensured that theoretical constructs such as the ZCA dimensions were contextualized and made observable in authentic classroom situations. This aligns with the perspective of Blanton and Kaput (2011), who emphasize that teachers' professional growth in algebraic thinking involves both conceptual understanding and the capacity to recognize algebraic reasoning in students' responses.

Overall, the study contributes to the growing body of research advocating for the integration of cognitive habit frameworks into algebra instruction. By linking algebraic demand types with habits of mind through the OSCA model, this work provides a structured approach for analyzing, designing, and reflecting on algebraic learning experiences. Future studies may further explore how teachers adapt these frameworks in diverse instructional contexts and how students' use of algebraic habits evolves over time.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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